The National Strategies
Primary

Problem solving with EYFS, Key Stage 1 and Key Stage 2 children

Finding all possibilities

Department for
Education

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Finding all possibilities

First published in 2010
Ref: 00346-2010PDF-EN-01

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Introduction

This resource focuses on problems that fall into the category 'Finding all possibilities'. To help children become good problem solvers it is important that they are taught and acquire essential skills and strategies and understand how and when to use them.

When solving 'Finding all possibilities' problems, the strategies children need to be able to draw on include:

- having a system for testing possibilities, e.g. start with a small number and build up to bigger numbers
- organising the recording of possibilities, e.g. make an ordered list or table and adapt it as more information is collected
- using a method of tracking what has been included and what has not to isolate relevant information
- having a way of checking for any repeats and deciding when all possibilities have been found.

There are two lessons for each age group, and a selection of activities for the Early Years Foundation Stage. Progression has been built across the two lessons in each year group so that the work in the second lesson builds on the work in the first, and also there is progression across the years.

You should adapt the problems for the needs of your children. For example, in the Year 1 problem that involves a robot moving down a path towards a target number, you could: change the numbers on the dice; alter the number of moves the robot makes; change the context or type of path.

Throughout the lessons, there are notes to indicate particular features that you might want to consider or look out for when working with children, indicated by the following symbol:

Foundation Stage

Activity 1: in the café

Early Years Foundation Stage early learning goal

- Use developing mathematical ideas and methods to solve practical problems.

Primary Framework objectives

- Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices.
- Sort objects, making choices and justifying decisions.
- Use developing mathematical ideas and methods to solve practical problems.

By the end of the lesson, children will be able to:

- recognise similarities and differences;
- sort objects, using a number of criteria;
- explain what they are thinking and doing.

Vocabulary

same sort different set

Necessary prior knowledge

- Colour recognition
- Language of similarity and difference
- Experience of playing in a 'house' situation

Resources

- Teddies or other soft toys
- Coloured crockery and cutlery

In the café

Role-play activity

Within role-play situations there are many opportunities for us to encourage the children to develop their problem-solving skills. This activity is in the context of role-play in the home or café, and involves the children in preparing the table for the teddies to have a meal. There will need to be a collection of soft toys and coloured crockery and cutlery from which they can choose. The adult's role may be as an observer of the children's play or 'in role' with the children.

Establish that the children understand the concept of setting the table. You may need to ask some of the children questions to get them started.

Children need to understand that problem solving involves choices so they have the opportunity to make decisions and justify them.

Q. Which teddies are having tea today? How many are there?
Q. Do you think there are enough plates for all the teddies to have tea?
Q. How can you be sure each teddy has a plate?
Q. How are you going to start setting the places?

Observe how the children tackle the problem.

Encourage the children to discuss with one another how the settings are the same or different. You may notice that some children match the colours systematically. 'I've given that teddy all the green ones.'

Q. What do you notice about the plates you have given the teddies?
e.g. Oh yes, those two teddies have both got a blue plate.
Yes, they are all different.
Q. What about the cups and saucers? Do they match? Are they different?

Create related problems and ask questions using other criteria to extend the children's thinking.
Q. Can you think of different ways of doing it?
Q. I think this teddy doesn't like red things. What can you do to help him have a setting he likes?
Q. Both these teddies want a yellow knife and fork. Can you sort that out for them?
Q. Can you make completely different settings for two teddies?
Q. These two teddies both want place settings that are only green and red. Can you make them both different?
Q. What would you do if another teddy came to the table? Is there enough room? Have you got enough cups?

How did the children tackle the problem? Did they understand the problem? Did they use a random approach? Did they use the words 'same/different'? Did any of them give each teddy all the same colour crockery and cutlery? Were they able to talk to others, including you, about what they were doing?

In the context of a related story or later discussion with a larger group of children about what they have been doing, it may be appropriate to model what some of the children did and draw out discussion about same/different.

Foundation Stage

Activity 2: working in the sand

Early Years Foundation Stage early learning goal

- Use developing mathematical ideas and methods to solve practical problems.

Primary Framework objectives

- Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices.
- Sort objects, making choices and justifying decisions.
- Use developing mathematical ideas and methods to solve practical problems.

By the end of the lesson, children will be able to:

- recognise similarities and differences;
- give examples that match a given statement and ones that don't;
- explain what they are thinking and doing.

Vocabulary

same sort how many? order
different set size match

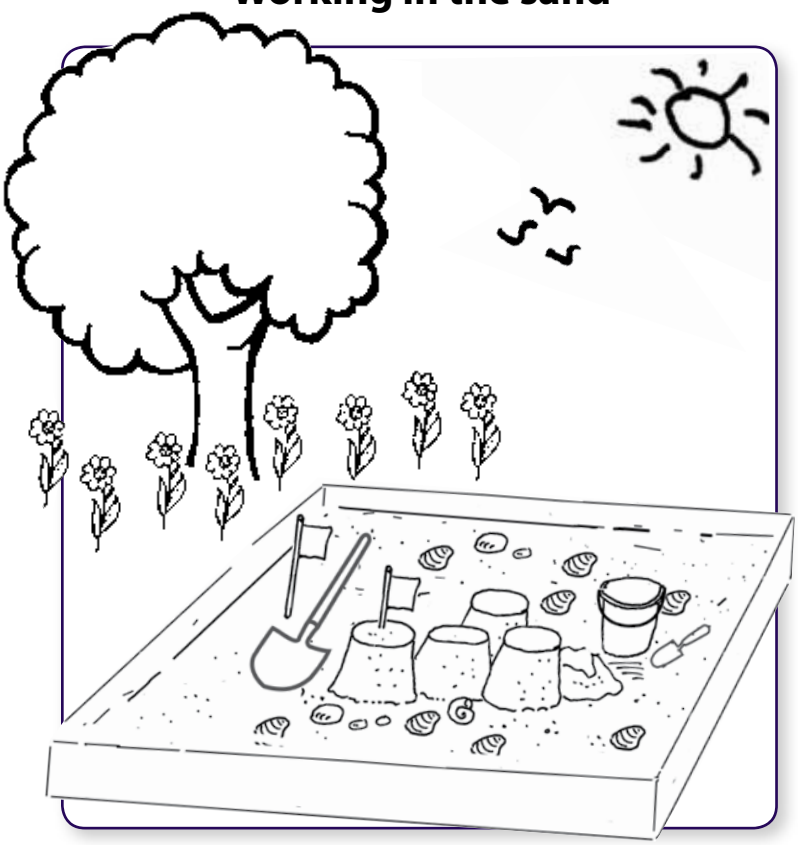
Necessary prior knowledge

- Language of similarity and difference

Resources

- Sand tray
- A collection of objects to hide in the sand

Working in the sand



Sand tray activity

This activity extends the use of the sand tray and may be done indoors or outside. You will need to decide whether the adult is to be an observer of how the children tackle the problem or an active player and prompter alongside them.

Put a collection of objects in the sand for the children to find and sort. The choice of objects may be linked to other learning in a variety of ways, for example, a current story (The Three Bears – teddy counters), an appropriate theme (The Zoo – animal counters), the role-play area (The Fruit Shop – plastic fruits) or another area of the classroom learning environment (Construction area – bricks, Autumn display – laminated leaves).

Perseverance and persistence are important problem-solving skills. 'I hid 10 in the sand. How many have you found? How many more do you need to find? Can you find the rest?'

Hide the objects for the children to find as they play in the sand.

Q. I've lost some things in the sand tray. Can you find them for me? Some are the same and some are different but they'll all be mixed up. See if you can sort them out.

It may be a good idea to have a collection of empty containers beside the sand tray into which the children can place the found objects as they sort them. For some children, it may be appropriate to extend the activities by hiding wooden or plastic numerals in the sand, giving children the opportunity to find, sort, match and also to order the found numbers.

If you are playing with the children, model showing one another what has been found and describing the objects using the vocabulary of same/different. 'Oh look, I've found a yellow banana too. It's the same as the one Jed found. Is it the same as your fruit, Kelly?'

Comparing two objects which have similarities and differences will help children to realise that there is more than one way to sort the collection of objects.

Encourage the children to compare the objects using different criteria such as colour, size or type, noticing similarities and differences. The children may have difficulty noticing objects which have similarities and differences, and may need your prompts: 'Yes, they are both the same because they're both apples, but what's different about them? Yes, Rashid, yours is a red apple and Lee's apple is green.'

Q. Lee, is your apple the same as Rashid's?

Q. You two have both found the same kind of creature; they're both snakes, but what's different about them?… Oh, Sally says her snake is longer than Harry's snake. Yes, Harry you're right, you noticed they're different as well because yours is green and Sally's is purple.

Encourage the children to use the empty containers to group the sorted objects and ask them to justify their groupings.

Q. So what's the same about all the things in this bucket?

The need to find all the objects gives the children a reason to count the ones they have found.

Q. How can we find out if we have found them all yet?

Some children may be able to sort the found objects further. They may be able to say, 'All these bricks are blue, but these are all small blue bricks and those are all big ones.'

In a later discussion with a larger group of children about what they have been doing, it may be appropriate to show how the children sorted the objects, drawing out what was the same and what was different.

If you have the use of an interactive whiteboard, you may want to create a screen with a collection of objects and sorting boxes into which the objects can be dragged and dropped.

Foundation Stage

Activity 3: railway track

Early Years Foundation Stage early learning goal

- Use developing mathematical ideas and methods to solve practical problems.

Primary Framework objectives

- Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices.
- Sort objects, making choices and justifying decisions.
- Use developing mathematical ideas and methods to solve practical problems.

By the end of the lesson, children will be able to:

- recognise similarities and differences;
- sort objects, using a number of criteria;
- explain what they are thinking and doing.

Vocabulary

same different

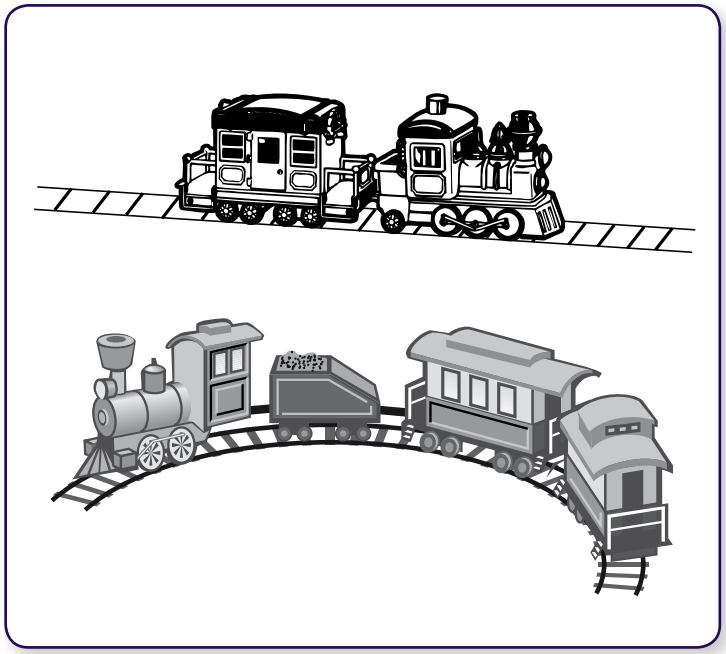
Necessary prior knowledge

- Language of similarity and difference

Resources

- Railway track and other related objects, e.g. trains, station, bridges

Railway track



Mini-world/construction activity

Mini-world and construction provide many opportunities for children to develop their problem-solving skills.

This activity will involve the children making two tracks to get from one point to another. It could be helpful to mark the beginning and end of the track with a station. The children will need a selection of straight and curved track and other related items such as bridges and level crossings. The adult's role may be as an observer of the children's play or as a player with the children. Ask the children to make two different tracks from one station to another.

Children need to understand that problem solving involves choices so they have the opportunity to make decisions and justify them.

It may be helpful to add to the real-life context by reading a story about railways and trains. You may need to ask some of the children questions to get them started.

Q. What sorts of pieces of track shall we use?
Q. What shall we put on our tracks?
Q. How can we make them different?

How did the children tackle the problem? Did they understand the problem? Did they use the words 'same/different'? Were they able to talk to others, including you, about what they were doing?

Observe how the children tackle the problem.
Encourage the children to discuss with one another how the tracks are the same or different.

Q. What do you notice about the two tracks you've made? Yes, that one's got lots of curved pieces and that one's got lots of straight pieces.
Q. Which track is longer?
Q. What else have you put on your tracks?
Q. What's the same about the two tracks?

It might be that both are made from wood, or both are made from plastic, or both have a bridge in them, or both have curved and straight tracks. They also start and finish at the same stations.

Create related problems and ask questions with other criteria to extend the children's thinking.
Q. Can you think of how we could make the tracks completely different?
Q. This train doesn't like too many curves. Where could we make some changes?
Q. This train can't go up hills. What might we need to change?
Q. Why might this track need to be longer than the other one? Can we add some extra stations on one track for people who only want to travel short distances? Which would be best? Why?

In the context of a related story or later discussion with a larger group of children about what they have been doing, it may be appropriate to sketch the tracks that some of the children made and discuss what is the same and what is different about them. If you have an interactive whiteboard or projection equipment, you could ask the children to take digital photographs of their track layouts and then choose some to prompt the discussion.

Year 1

Lesson 1: lollipops

Primary Framework objectives

- Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions.
- Answer a question by selecting and using suitable equipment, and sorting information, shapes or objects; display results using tables and pictures.

By the end of the lesson, children will be able to:

- recognise that there is more than one possible answer to a problem;
- explain how answers differ and to record different answers in a list.

Vocabulary

pence total cost list coin buy pay same different

Necessary prior knowledge

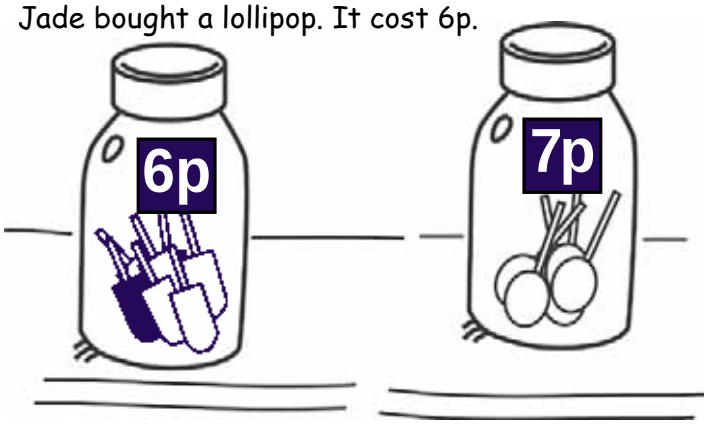
- Recognise coin values and understand the order of the values
- Add more than two numbers, 5 or less
- Count in ones and twos

Resources

- Resource sheet
- Jars of lollipops or other items labelled with '4p each', '6p each' and '7p each'
- Puppet
- Large coins
- Real coins (1p, 2p and 5p)
- Strips of card with double-sided tape stuck on the back
- Sheet of A2 paper (optional)

Lollipops

Jade bought a lollipop. It cost 6p.



She paid for it exactly. Which coins did she use?

There are 5 different ways to do it.
Find as many as you can.

What if the lollipop cost 7p?

Main teaching activity

Set the context for the problem by showing the class several jars of lollipops with '4p each', '6p each' and '7p each' labels or items from the class shops, similarly labelled. Explain that anyone who wants to buy a lollipop from the jar must pay the exact amount for it.

Introduce a puppet named Jade (or any familiar class puppet). Explain to the children that they are going to solve problems today to help Jade to buy some lollipops.

Draw the children's attention to the jar labelled '4p each'.
Q. If Jade wants to buy one lollipop out of this jar how much money will she need?
Identify that she will need coins. Agree that she needs 4p.
Q. What will Jade use to pay for the lollipop?
Q. What coins might she use?
As the children respond, put large 1p and 2p coins on the board.
Q. Why wouldn't she use a 5p, 10p, 20p or 50p coin?
Agree that these are more than 4p.

Invite a child to come and give you 4p for a lollipop. Show the children how to record the solution, for example by drawing four 1p coins (or displaying large coins), and recording:
1p + 1p + 1p + 1p = 4p

If the children only suggest 1p coins, hold up a 2p coin and ask how much more money is needed to make 4p.

Stress the importance of checking the total, for example by keeping a tally on your fingers.

Some children will not be aware that there is more than one answer. It is important for them to understand that the cost is the same, but that you are using different coins to pay and there are different ways to do this.

Q. Is there another way to pay for a lollipop?
Ask the children to work in pairs to find different ways to make 4 pence. If children suggest examples where giving change is involved, remind them that in this situation they have been asked to find the exact amount.

Drawing together

Draw the class together to share solutions.
With the children, check that no answer has been repeated.
Q. Why are these answers different? Why are these answers the same?

To help the children 'see' the answer, attach the coins to the board, then record the number statement alongside.

Draw out that 1p + 1p + 2p uses the same coins as 1p + 2p + 1p and 2p + 1p + 1p.

Helping children to recognise the same and different answers helps them to be more systematic. Making a list is one way that children can be helped to see whether all possibilities have been found.

Make a list of the solutions:
1p + 1p + 1p + 1p = 4p
1p + 1p + 2p = 4p
2p + 2p = 4p

Tell the children that you have recorded the different answers in a list.
Q. How many different answers were there to this problem? Are there any more answers?
Agree that there are three answers.

Draw the class's attention to the jar labelled '6p each'.
Q. If Jade wants to buy a lollipop out of this jar, how can she pay?
Q. How many different ways do you think Jade can pay? Why?
Explain that Jade wants to know all of the different ways she would be able to pay for the lollipop.
Ask everyone in the class to find a way to make a total of 6 pence.
Explain that the class will regroup to make a list of the different answers they have found.

Pursue the conversation to assess the children's understanding using questions such as: 'How do you know that?', 'Why do you think that?'

On an A2 sheet of paper write the title 'Ways to make 6p'. Explain that you will record the list of all ways to make 6p.

Ask the children to each find one way to make 6 pence using the real coins. Encourage them to find a way that is different to their neighbours'.
When they have laid out their 6 pence amounts they should stick them onto a strip of card with double-sided sticky tape as a record.

Plenary

Ask the children to bring their strips of card and coins to the plenary. Remind the children that the problem was to find all of the ways in which Jade could pay 6p for her lollipop. Say that they now need to make a list of the different ways and check whether the answers are the same or different.

Invite a child to put their solution onto the paper.
Q. Does this equal 6p? How can we check?

Invite other children to add their solutions to the list. Each time ask children to check that the amount equals 6 pence and that it is different to the other solutions.
Q. Does this solution equal 6 pence?
Q. How could we change it so that it does equal 6 pence?
Q. Have we had this solution already? How do you know?
Q. How is this solution different to this one?

Show how to make a written record of the five solutions:
1p + 1p + 1p + 1p + 1p + 1p = 6p
1p + 1p + 1p + 1p + 2p = 6p
1p + 1p + 2p + 2p = 6p
2p + 2p + 2p = 6p
1p + 5p = 6p

You might want to model written recording, first drawing the coins, then writing a number sentence alongside.

Conclude the lesson by referring back to the original problem.

Q. How many different ways of paying 6p for the lollipop did we find?

Q. Which way uses the most coins?

Agree that it is when we pay with 1p coins only.

Q. How many 1p coins did we use?

Agree that it was six.

Q. Which way uses the fewest coins?

Agree that it is when we use a 5p coin and a 1p coin.

Q. If the lollipop cost 8p, how many 1p coins would we use?

Q. Could we use 2p, 5p or 10p coins?

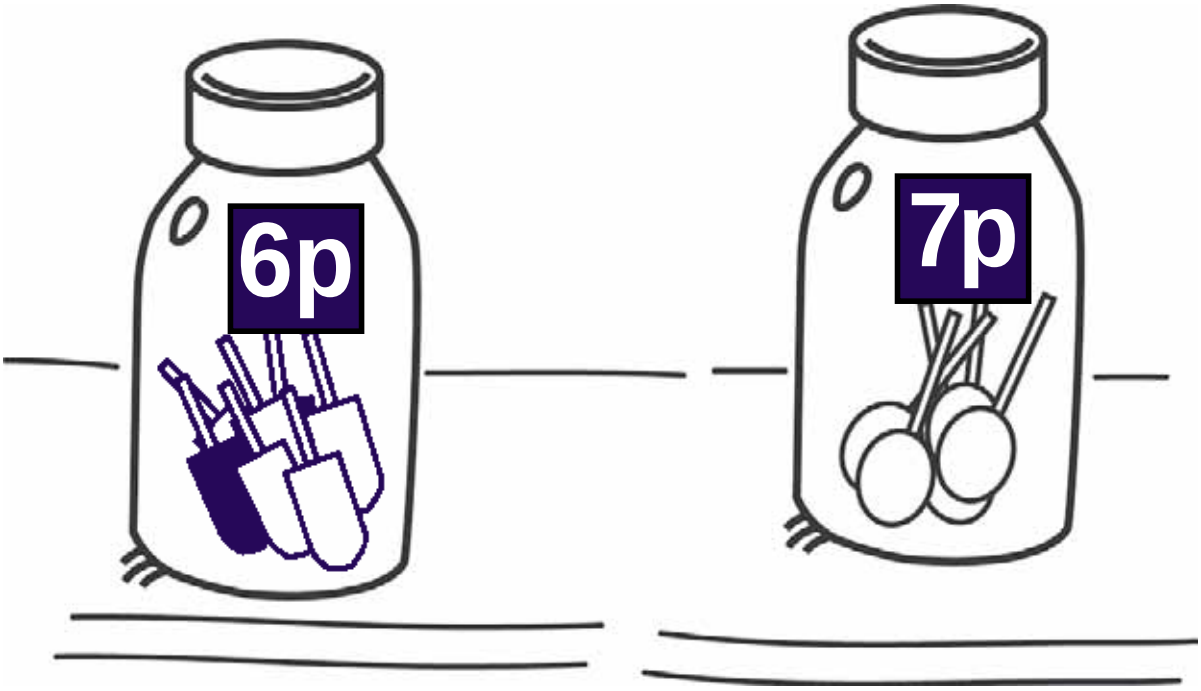
Agree that 2p and 5p coins could be used but 10p is too much.

Q. If we used a 5p coin, how much more money is needed? What coins could we use to make that amount?

Year 1, Lesson 1: resource sheet

Lollipops

Jade bought a lollipop. It cost 6p.



She paid for it exactly. Which coins did she use?

There are 5 different ways to do it.
Find as many as you can.

What if the lollipop cost 7p?

Year 1

Lesson 2: down the path

Primary Framework objectives

- Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions.
- Answer a question by selecting and using suitable equipment, and sorting information, shapes or objects; display results using tables and pictures.

By the end of the lesson, children will be able to:

- explain how answers are different and to record different answers in a list;
- begin to have a system for ordering a list of possibilities.

Vocabulary

list count on

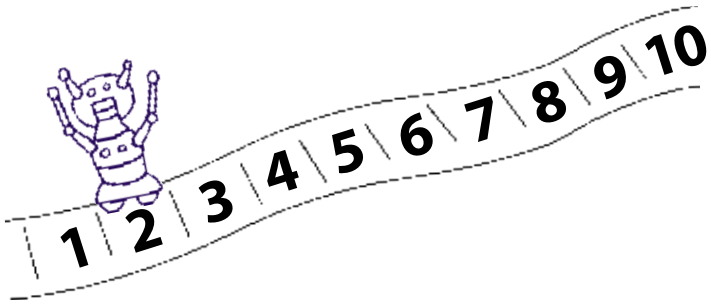
Necessary prior knowledge

- Adding two numbers
- Counting on
- Pairs of numbers that make 6
- Playing a track type game

Resources

- Resource sheets 1 and 2
- Large version and individual copies of resource sheet 3
- 1–10 floor number track
- Moveable robot or object
- Objects for moving along individual
  tracks on resource sheet 1
- Two large demonstration dice
- Individual dice

Down the path



The robot is on 2.
You roll a 1 to 6 dice.
After 2 moves it lands on 8.

Find all the different ways the robot can do this.

Now think of other questions you could ask.

Main teaching activity

Sit the class around a large-scale number track (the path) numbered 1 to 10 and a robot/object to move along it. Give each pair of children a copy of Resource Sheet 1, two dice and an object to move along one of the tracks in front of them. Ensure that the children understand the idea of the game – to roll the dice and to move the object along the track the number of steps rolled.

Q. If the robot is on 5 and I want the robot to move onto 7 next, what number would I now be hoping to roll on the dice?

This is a good opportunity to model recording as a number sentence: 5 + 2 = 7

Take responses and try them out. Agree that two more steps would take the robot to 7. You may need to repeat this process several times to ensure that the children are used to reaching a target number, rather than counting on the number stated. Some may want to count on another 7 rather than visualise what is needed to get to 7.

As they work, observe how well children count on. Intervene to model the language of adding the two scores together to reach a position.

'You rolled a 4 and then you rolled a 3, so 4 steps and then 3 steps meant you arrived at 7.'

Ask the children to work in pairs to find other pairs of numbers that would enable the robot to move from the start to 7 in two moves. Ask them to turn their dice to show the two numbers that might be rolled.

Drawing together

Q. What two numbers did you turn your dice to?
Model the recording, e.g. 3 + 4 = 7
Q. What is the biggest number you could use? What would you need to get next?
Record 6 + 1 = 7
Q. What's the second biggest number you could use? What would you need to use next?
Record 5 + 2 = 7

It is important to make connections to previous lessons. You may want to show the list from yesterday here.

This is a good opportunity to model systematic working.

Continue until all the solutions are recorded.
Discuss that 2 + 5 = 7 and 5 + 2 = 7 are similar but the order that the numbers come up on the dice is different.

Give out Resource Sheet 2 and explain that the robot is now starting on the number 2 and you want it to get to the number 8, but it must be done with two rolls of the dice. Explain that there is more than one way to do this and we need to find all of the different ways. Remind them of the list that you have just made and say that this is a useful way to record the different ways. Show a large version of the Activity Sheet to show how the different possible answers are going to be listed.

Pursue the conversation to assess the children's understanding such as 'How do you know?...' and 'Why do you think that?'

Demonstrate by using two large dice, one for the first throw and one for the second throw. Tell the class that the first throw might be a 4 (showing a 4 on the large dice). Set the robot to move forward 4 from 2 to land on 6.

Q. So what would my second throw have to be?

Establish that it would be 2. Show the 2 on the second dice. Test by getting the robot to move forward 2.

Q. How many moves did it take to get to 8? Establish that 6 moves are needed, and that is what the problem is asking. In other words 'How can we make 6 with two dice?' Say that we could also record this as 4 + 2 = 6.

While children are engaged in this activity, look for examples that show systematic working.

Ask the children to work in pairs to place their object on number 2, and then turn (not roll) their two dice as a pair to agree what their two dice rolls would need to be to get to 8. They should record this solution on the Activity Sheet. They may wish to circle the numbers they landed on, on Resource Sheet 1. You may need to remind them that a total of 6 is needed.

Drawing together

Take children's responses on any pairs they have found. Invite two children out to the front to show two dice scores with the large dice.

Q. If we want to make a list of the different solutions, what would be a good one to start with?

Agree that starting with the smallest or largest number is helpful.

Q. How is this answer different to this one?

Model how to record in a systematic way on the large version of resource sheet 3. Ask the children to work in pairs to find other pairs of dice scores that make 6 and record them on the resource sheet 3.

Plenary

Bring the children together, along with their lists, to see whether different ways to solve the problem have been found.

Test answers by moving the robot along the track using the children's answers. Identify different answers and compile a class list in a systematic order:

1 and 5
2 and 4
3 and 3
4 and 2
5 and 1.

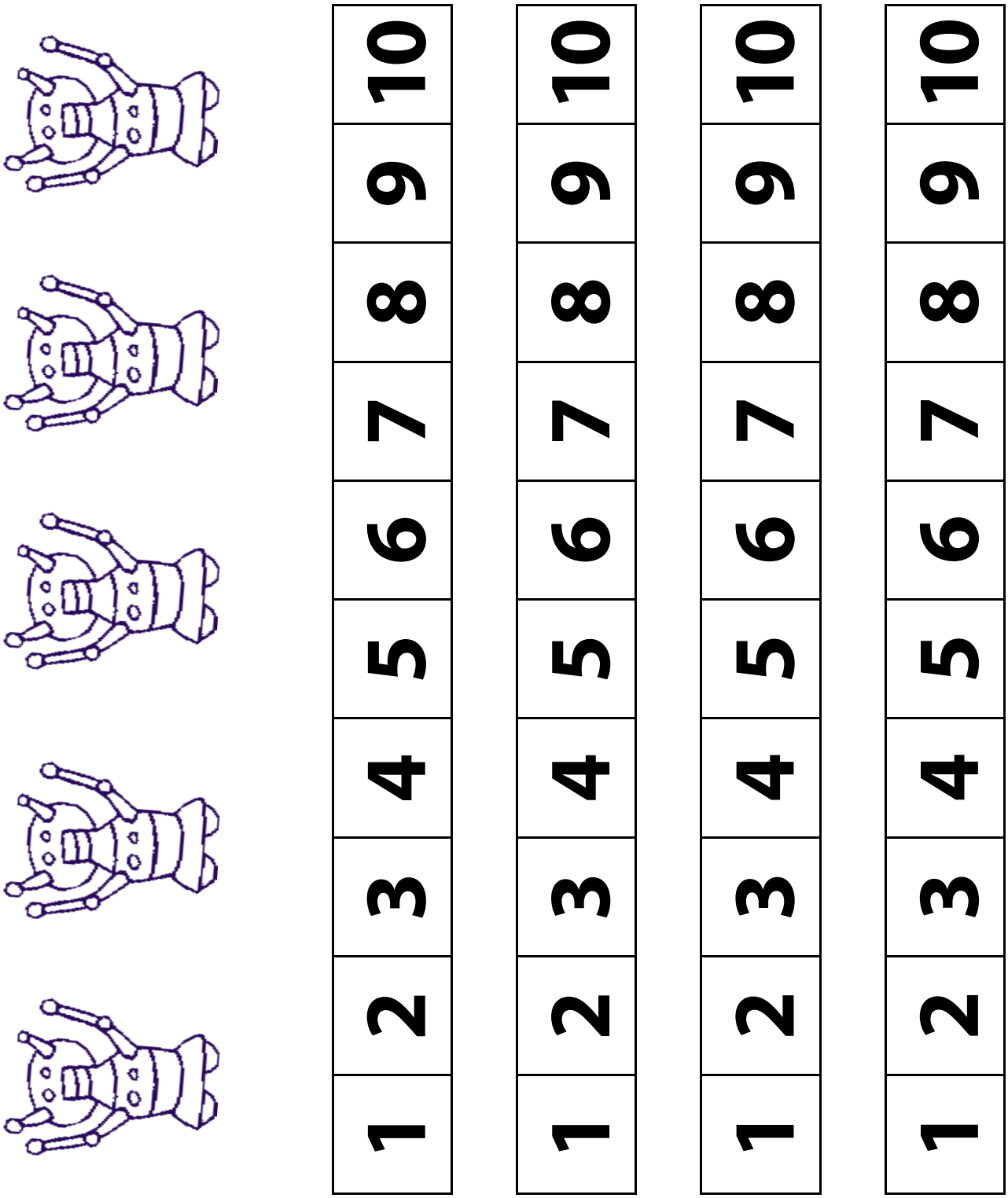
Point out the pairs of answers:

1 and 5	2 and 4
5 and 1	4 and 2

Draw out the systematic order of the list as a teaching point for future problem-solving activities.

Discuss how the same pairs of numbers were rolled but in a different order. Conclude the lesson by referring back to the problem and agreeing that there are five different ways to solve the problem.

Year 1, Lesson 2: resource sheet 1



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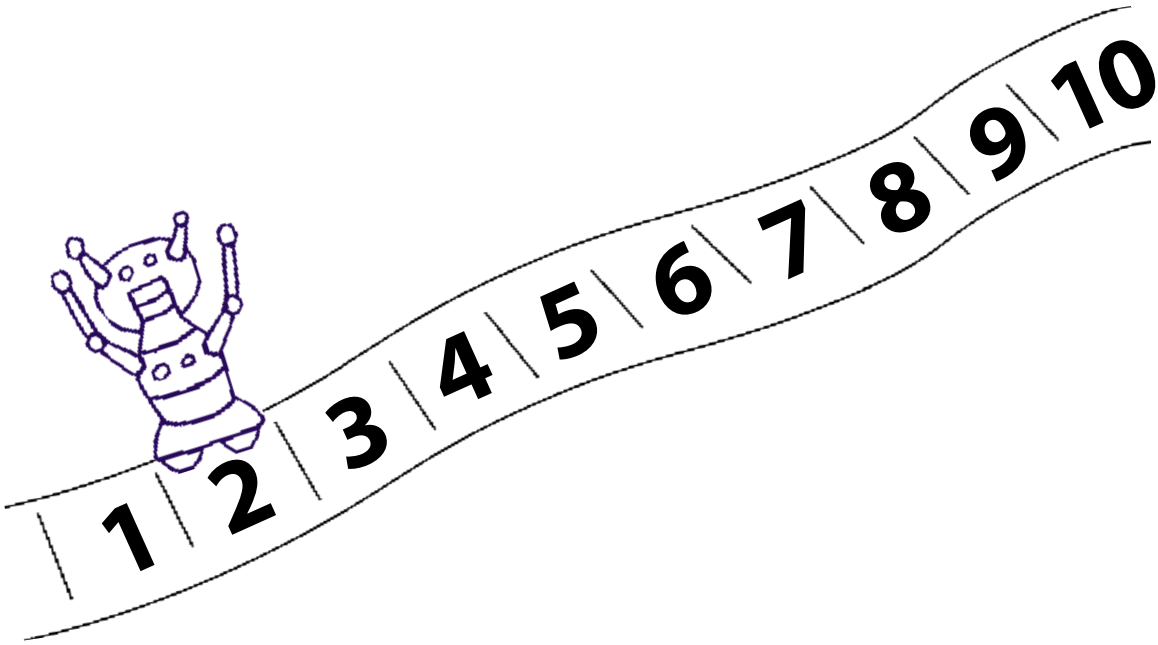
Year 1, Lesson 2: resource sheet 2

Down the path



The robot is on 2.
You roll a 1 to 6 dice.
After 2 moves it lands on 8.

Find all the different ways the robot can do this.

Now think of other questions you could ask.

Year 1, Lesson 2: resource sheet 3

Ways to land on 8, in 2 moves

move 1	move 2

Year 2

Lesson 1: Maisie and the maze

Primary Framework objectives

- Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions.
- Answer a question by selecting and using suitable equipment, and sorting information, shapes or objects; display results using tables and pictures.

By the end of the lesson, children will be able to:

- begin to use a systematic way to solve a problem involving finding all possibilities;
- create a clear list of possibilities.

Vocabulary

list possibilities systematically

Necessary prior knowledge

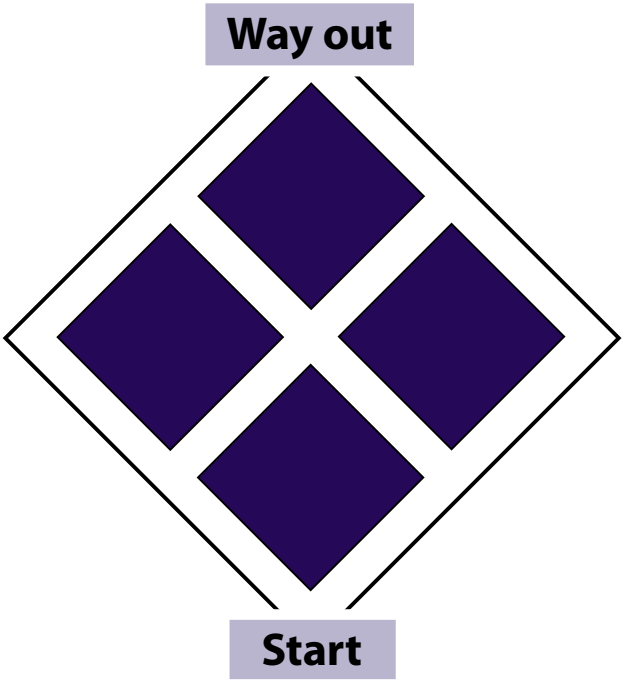
- Directions: right, left

Resources

- IWB version of Maisie and the maze
- Resource sheet 2 providing copies of the maze
- Maze drawn five or six times on the playground with chalk

Maisie and the maze

Maisie explored the maze.
She always went forward.



How many different ways are there for Maisie to go from the start to the way out?

Main teaching activity

Show the children a large version of the problem and explain that this is the maze which Maisie is going to explore. She is trying to find the way out. She must always go forward, never back.

Take the children outside and show them one maze (previously drawn in chalk on the playground). Show them the start and the way out. Ask the children to talk to a partner about which way Maisie could go, to get to the way out.

Ask a child to walk on the paths, keeping to the rule of always moving forward, and find the way out. Using other identical mazes drawn on the playground, let the children have a go at being Maisie.

Drawing together

Come back inside.
Ask a child to draw, on the projected image, the route he or she decided on.

Q. Was she always going forwards?
Agree that she was, and that this is a rule in this problem.
Q. Are there any other ways she could get to the way out?
Give the children copies of resource sheet 2 and ask them in pairs to find another route and draw it on their maze. Say that although there are eight mazes, this doesn't mean there are eight different routes through the maze, but these allow them to experiment and not have to draw all their routes on the same maze. Some may need to find the routes by using the playground mazes supported by a TA if available.

Ask each pair to find other pairs of children and look at their routes.

Q. How many different routes are there?
Gather some answers and record these on the interactive whiteboard.

Q. How can we record our routes?
Discuss that we could draw the routes on the maze and demonstrate that this could be hard to see on one sheet.

Q. Would we be able to count the separate routes? Agree that this would be difficult since they would overlap.

Q. Are there other ways of recording the routes so that someone else could read them and understand them?
Lead the children to using left and right.

Discussing how we know whether we have found all the possibilities can lead children to understand the need for recording in an efficient way.

Do some children have difficulty leaving the visual recording and using L, R? Is this because they have trouble with left and right or is it the need to see the route on the maze rather than visualise and record?

Demonstrate the first route by recording:
L, R, L, R as you draw it on the maze.

Ask a child to show their route on the maze and then record it using L and R.
Tell the children that we are going to make a list of the routes. Demonstrate this by asking another child to record another route under the first.
Ask the children to now record all their routes using this method. Recording L, L, R, R may cause a problem since we don't actually turn left on the second left, we keep left. This may need explaining.

Drawing together

When they have found all the routes they can, ask:
Q. How many different routes are there?
Gather answers and record them on the whiteboard.

Take one set of routes and discuss it.

Q. Are any routes repeated? Have you checked?
Establish the importance of checking.

Q. What do you notice about the answers?
Demonstrate that there are always 2 lefts and 2 rights. If they don't notice this, ask how many left and rights there are.

Q. Why are there never 3 rights and 1 left?
Demonstrate that if there were, Maisie would be going back.

Q. How do we know all the routes are there?
Q. How do we know that we haven't missed any?

Lead the children to understand the need to work and record systematically. Demonstrate by drawing the routes in a haphazard way that we can miss some.

Q. Where could we start so that we work systematically?

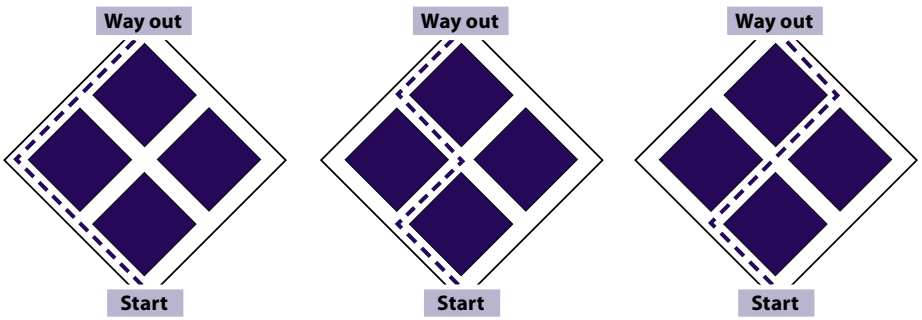
Agree that when we start from the bottom of the maze we can either go left or right and so we could start by going left and finding those routes. We could then go right and find those routes. Some children may see the symmetry and realise that there will be the same number.

Finding patterns can help us solve problems.

Ask the children to find all the routes systematically and record them systematically.

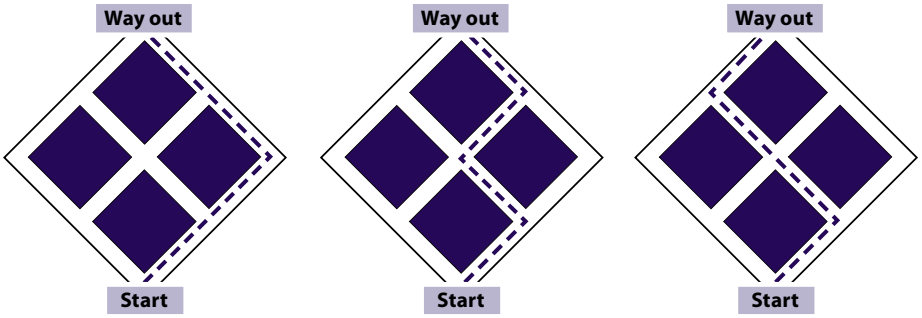
Drawing together

Q. How many routes are there, starting by going left?
Demonstrate that there are 3.



Q. How many routes are there starting by going right?

Ascertain that there are 3.



Draw out the need for a clear list so that we can count the number of routes. Ask the children who went back to the picture to think about whether they could have used their list.

Plenary

Ask the children to work in pairs to answer:

Q. How many routes would there be if she always had to change direction and couldn't go left, left, or right, right?

'What if?' questions provide useful extensions to problems.

Encourage them to look at their lists and discuss whether these can help.

Look at one list and find the possibilities. Lead them to understand that we can use our recorded answers to answer other questions.

Note whether the children use their recording to find the answer or go back to the picture.

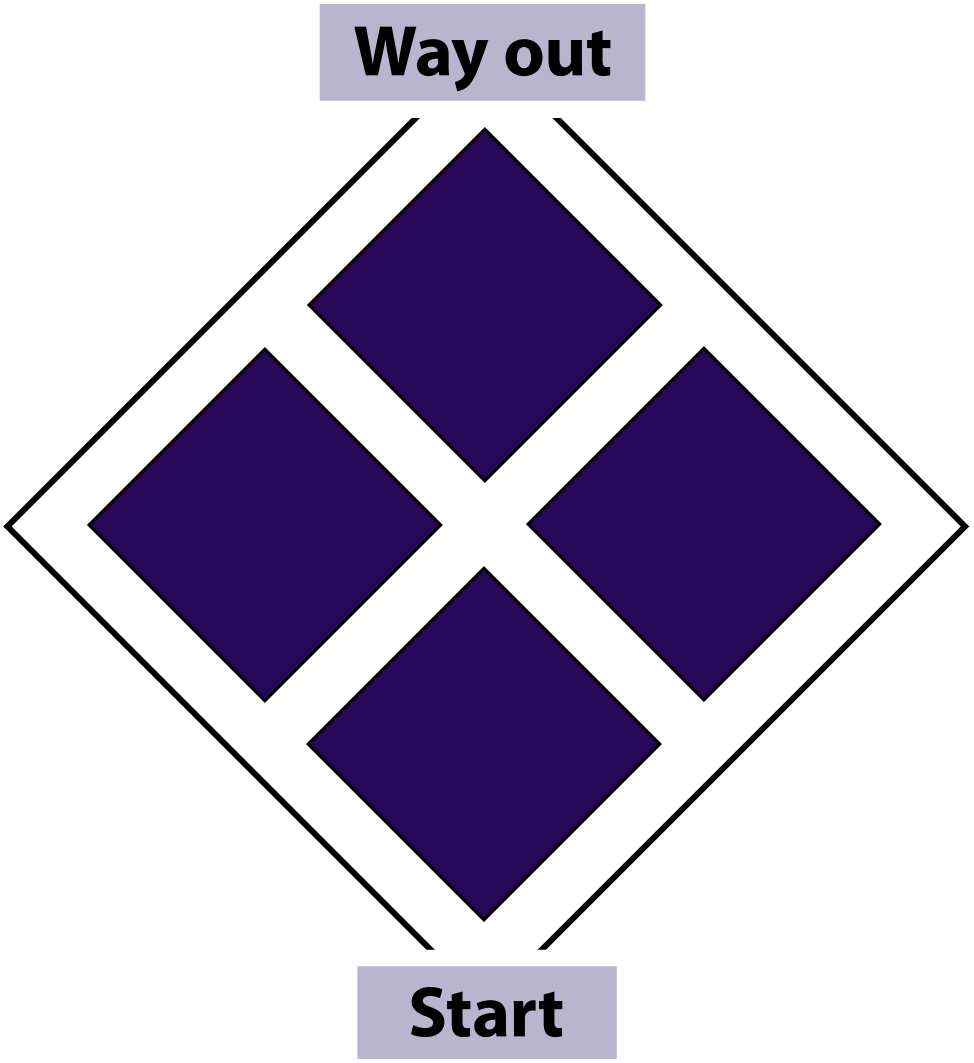
Discuss the important points:

- recording systematically;
- checking that we have not repeated any answers;
- checking that we haven't missed answers.

Year 2, Lesson 1: resource sheet 1

Maisie and the maze

Maisie explored the maze.
She always went forward.



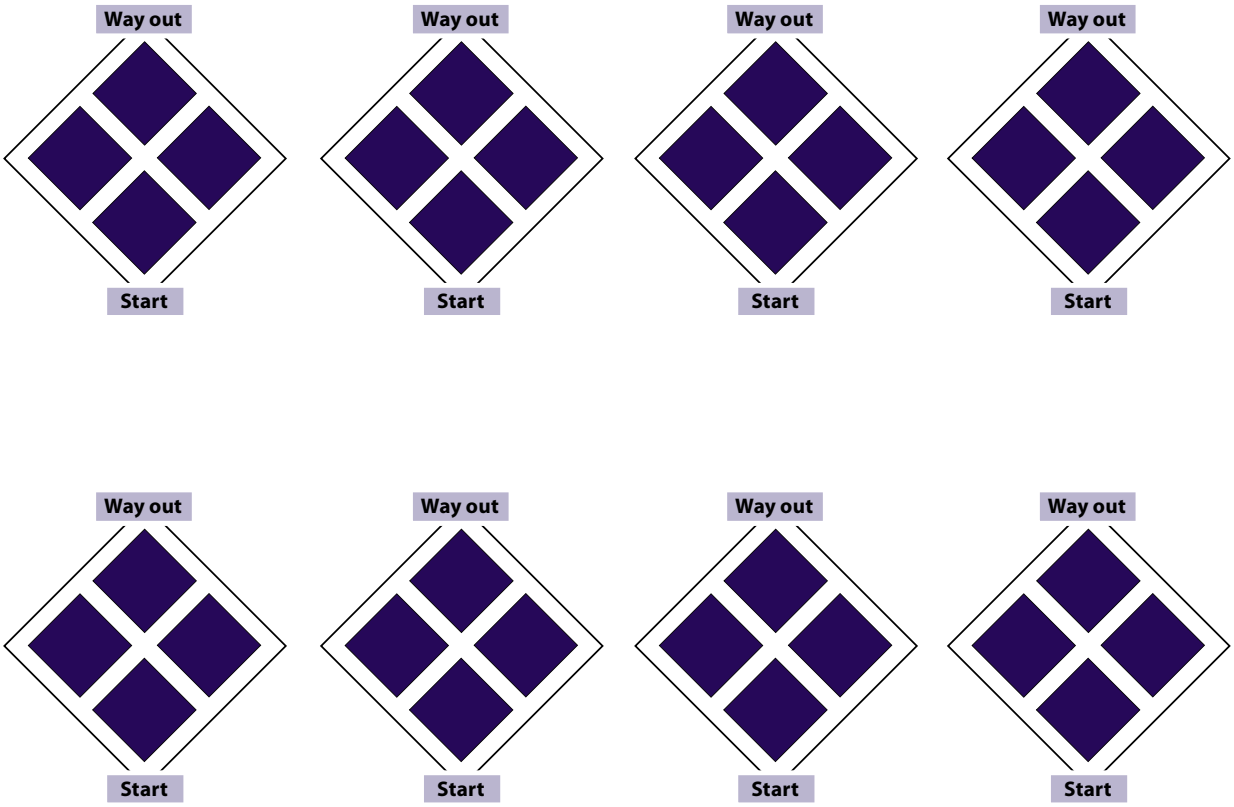
How many different ways are there for Maisie to go from the start to the way out?

Year 2, Lesson 1: resource sheet 2

Maisie and the maze

Maisie explored the maze.

She always went forward.



How many different ways are there for Maisie to go from the start to the way out?

Year 2

Lesson 2: line of symmetry

Primary Framework objectives

- Describe patterns and relationships involving numbers or shapes, make predictions and test these with examples.
- Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem.

By the end of the lesson, children will be able to:

- find and record all possibilities for a problem in a list systematically.

Vocabulary

list possibilities systematically

Necessary prior knowledge

- Symmetry

Resources

- IWB version of counters and resource sheet 1.
- Coloured squares – some red, some blue and some green
- Squared paper
- Large mirror
- Small mirrors

Line of symmetry

You need:
some squared paper, a red pen, a green pen and a blue pen.

Gopal had six squares: two red, two green, two blue. He put them in a line.
The squares made a symmetrical pattern.

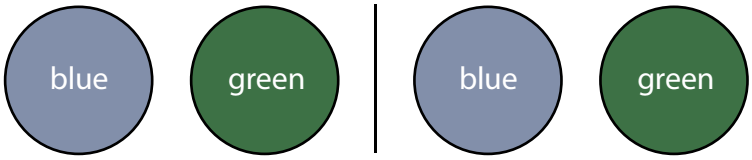
red	**blue**	**green**	**green**	**blue**	**red**

Arrange six squares in a line.
Make two squares red, two green and two blue.
Make the line of squares symmetrical.

How many different lines can you make like this?

Main teaching activity

Using counters on the interactive whiteboard show a line:



Draw a vertical line between the two middle counters. Ask the children to discuss with a partner whether the pattern made by counters is symmetrical about this line.

It is key to this problem that children understand symmetry in this context.

Gather some answers.
Agree that it is not.
Q. Why not?

Demonstrate with a mirror that it is not.
Q. How could we make it symmetrical?
Ask a child to move the counters to make it symmetrical.

Show the children the IWB version of the problem. Read it together.
Invite children to underline or highlight important words and numbers.
Q. What did Gopal have?

Establishing the rules shows whether the children have understood the problem.

Establish that he had 2 red, 2 green and 2 blue squares.
Q. What did he have to do?
Agree that he had to put them in a line and the line had to be symmetrical as we did on the interactive whiteboard with the counters.

Ask the children to work in pairs to solve the problem. Have coloured squares available, squared paper and mirrors. If children can't think of how to start, show them the squares and ask whether they can make a line which is symmetrical. Can they then make another line?

Drawing together

Q. How did we record our answers yesterday? What was important?
Remind children of how they thought of a quicker and more efficient way of recording using left and right and that they recorded systematically.

Q. Why did we do this?
Agree that it helped us to find all the possibilities and we were less likely to miss any.

Ask the children to remember this when they are doing today's problem and, in pairs, to carry on solving the problem.

Drawing together

Ask some children to describe how they solved the problem.

Q. If the line has to be symmetrical how many of each colour will be in half of the line?

Agree that there will be one of each colour in half of the line and the rest in the other half of the line.

Encourage children to think about the different methods used. Did some children realise that their method was not as efficient as others?

Q. If we start with red, how many different lines will there be?
Agree that there will be two.
The first could be: red, green, blue.

Q. Where do the other 3 counters go?
Ask a child to put them on the interactive whiteboard: red, green, blue, blue, green, red. Ask a child to check it with the mirror.

Q. How could we move the counters to get another way?
Agree that the green and blue counters could be swapped. Help a child to do this on the interactive whiteboard: red, blue, green, green, blue, red.
You could ask a child to check it with the mirror.

Use a child's recording to show to the rest of the class.
Remind them that we are making a list of each possibility:
red, green, blue, blue, green, red
red, blue, green, green, blue, red

This establishes a starting point so that we can be systematic. Tell the children that we could have started with any of the colours.

Q. If we start with green, how many different lines will there be?
Ask children to discuss this with a partner. Ask them to write down the possibilities.

Collect answers.
Agree that there will be 2 and model recording as was done with red as the first counter:
green, red, blue, blue, red, green
green, blue, red, red, blue, green

Q. If we start with blue, how many different lines will there be?
Ask the children to find the possibilities with their partner.
Agree that there will be two possibilities.

Do most children realise that there will be 2 possibilities since there were 2 with red at the beginning and 2 with green at the beginning?

Draw out that there were 2 starting with red and then with green so there should be 2 starting with blue: blue, green, red, red, green, blue, and blue, red, green, green, red, blue.
Show this on the interactive whiteboard.

Add the last two lines to the list.
Ask the children to look at the list and tell their partner what they notice.

Do they see patterns?
Do they see that all the colours are in all positions?

Talk about the lists. Draw out the positions of each colour in the lines.

Q. What do we need to check?
Q. Have we got them all?

Do the children think we have? Ask, 'How do you know?' How do they explain their reasons?

Plenary

Show the different ways of recording.

There may be:
coloured squares placed on the table, coloured-in squares on squared paper, a written form red, blue, etc., a shortened written form, e.g. r, b, g
Q. Which is most efficient?
Q. Which takes a long time to record?
Q. What if I gave Gopal 2 yellow squares? How many possibilities would there be?
Show an image of all the possibilities with one line covered up.

'What if?' questions provide useful extensions to problems.

It is not expected that children will find the actual answer but talk about how many there might be.

Q. Which line is covered?
Q. How do you know?

If the children find it hard to find the missing one, ask questions such as 'Have we got all the lines that start with red?'

Year 2, Lesson 2: resource sheet

Line of symmetry

You need:
some squared paper, a red pen, a green pen and a blue pen.

Gopal had six squares: two red, two green, two blue. He put them in a line.
The squares made a symmetrical pattern.

red	**blue**	**green**	**green**	**blue**	**red**

Arrange six squares in a line.
Make two squares red, two green and two blue.
Make the line of squares symmetrical.

How many different lines can you make like this?

Year 3

Lesson 1: fireworks

Primary Framework objectives

- Identify patterns and relationships involving numbers or shapes, and use these to solve problems.
- Follow a line of enquiry by deciding what information is important; make and use lists, tables and graphs to organise and interpret the information.

By the end of the lesson, children will be able to:

- prove that they have found all possible answers to a problem by generating a list;
- check that their solution fulfils the criterion.

Vocabulary

odd even multiple times continue number pairs total altogether equals list rule

Necessary prior knowledge

- Recall of multiples of 3 and 4
- Addition facts within 20

Resources

- IWB version of the Fireworks problem (resource sheet 1)
- Number lines
- Whiteboards

Fireworks

Emma had some fireworks.
Some made 3 stars.
Some made 4 stars.

Altogether Emma's fireworks made 19 stars. How many of them made 3 stars?
Find two different answers.

What if Emma's fireworks made 25 stars?
Find two different answers.

Main teaching activity

Tell the children, 'We are going to solve a problem with more than one answer.
We need to find all of the answers. We need to think about how we are going to record our working, to know when we have found all of our answers'.
Ask the children to list the first six multiples in the 5 × table on the left of their whiteboards. Ask them to circle the largest even number and the smallest even number.
Now ask the children to list the first ten multiples in the 3 × table on the right of their whiteboards.
Ask the children to circle all even multiples in both lists above 20 and share their responses with a partner.
Next ask the children to circle all the odd multiples in both lists below 20 and share their responses with a partner.

Asking the children to list and circle a multiple or multiples that fulfil a given criterion will encourage and practise the skills that they will need to apply later.

Q. Which of these odd multiples below 20 is a two-digit number?
Q. What can you tell me about this number?
Establish that it is odd, a multiple of 3 and of 5, below 20 and a two-digit number and that these are the rules that this number fits.

Show the IWB version of the Fireworks problem. The use of manipulatives such as cards/fireworks with 3 stars, 4 stars, number lines, 20 beadstrings, may help some children access the problem.
Set a context for the problem. 'Emma's brother is 19 today and she wants to make a firework display with that number of stars, to celebrate.'
Q. Can you describe the problem to a partner in your own words?
Q. What do you already know? How many different sorts of fireworks has she got? How many stars does Emma want in her display?
Q. What have we got to find out?

Ask the children to talk to their partner about how they might start to work out the problem.
Q. Where are you going to start?
Q. How many 3-star fireworks could Emma set off?
Q. How far do we need to go?

Give the children just a few minutes for this.

At this point allow children free choice of recording.

Establish that they don't need to set off more than six 3-star fireworks as this would produce 18 stars, and so one more 3-star firework would produce too many stars.
Ask the children to work in pairs to solve the problem. If some children are unsure at this point and are working randomly, suggest starting by listing multiples of 3 to find the numbers of stars. Some children may solve the problem quickly through trial and improvement. If so, ask, 'Emma's sister is having her 25th birthday soon. How many 3-star and 4-star fireworks might she use for her display?' Without telling them that there are two solutions ask them to convince you that they have found them all.

Make a note of the children using a systematic approach and the range and efficiency of recording being used.

Drawing together

Q. How did you find the answer?
Q. Does your recording help you? How?
Q. Could someone else understand the way you have worked it out?
Discuss the different strategies and methods of recording.
Remind the children that they will need to know when they have found all possible solutions.

Use a child's example that shows systematic listing and ask that child to explain their thinking.

Pursue this conversation to assess the children's understanding using questions such as 'How do you know?' and 'Why do you think that?'

Draw out that listing allows us to check and convince others that we have found all of the answers.
On the board present the two lists together side by side.
Establish that they need to look for a pair of numbers from two lists, one from each, with a total of 19.
Ask for children's responses to this.

The expected outcomes are more concerned with using lists to prove that they have found all of the solutions to a problem, rather than solving the problem only through listing.

Establish that 5 (3-star fireworks) and 1 (4-star firework) = 19
1 (3-star firework) and 4 (4-star fireworks) = 19

Q. Have we solved our problem?
Q. What did Emma want?
Draw out the criteria from the children.
Reread the problem, checking against all of the criteria.

Ask the children to solve the second problem on the interactive whiteboard, i.e. to find the number of 3-star fireworks and 4-star fireworks if Emma's fireworks made 25 stars.

Q. What can we do to help?
Draw out that they can extend their lists and look for pairs with a total of 20. Ask them to do this, and then to pose their own question about Emma's fireworks. They should work in pairs to answer each other's questions.

Plenary

Collect several of the children's own questions and discuss them together.

Q. If Emma made a display with two 3-star and three 4-star fireworks, how many stars would she have?
Q. What if she lit five 3-star and two 4-star fireworks?
Q. What if it was her 21st birthday?

'What if?' questions provide useful extensions to problems.

Encourage children to decide if they need to extend their lists and explain their reasoning. Establish that three 3-star fireworks and three 4-star fireworks, or seven 3-star fireworks, would give 21 stars.

Ask children questions that will encourage them to use their lists of multiples to find pairs that satisfy the criteria.

Year 3, Lesson 1: resource sheet 1

Fireworks

Emma had some fireworks.
Some made 3 stars.
Some made 4 stars.

Altogether Emma's fireworks made 19 stars. How many of them made 3 stars?
Find two different answers.

What if Emma's fireworks made 25 stars?
Find two different answers.

Year 3

Lesson 2 : Susie and the snake

Primary Framework objectives

- Identify patterns and relationships involving numbers or shapes, and use these to solve problems.
- Follow a line of enquiry by deciding what information is important; make and use lists, tables and graphs to organise and interpret the information.

By the end of the lesson, children will be able to:

- find, and prove, that they have found all possible answers to a problem by generating a list and checking that their solution fulfils the criterion.

Vocabulary

lots of total groups of altogether multiple of equals time

Necessary prior knowledge

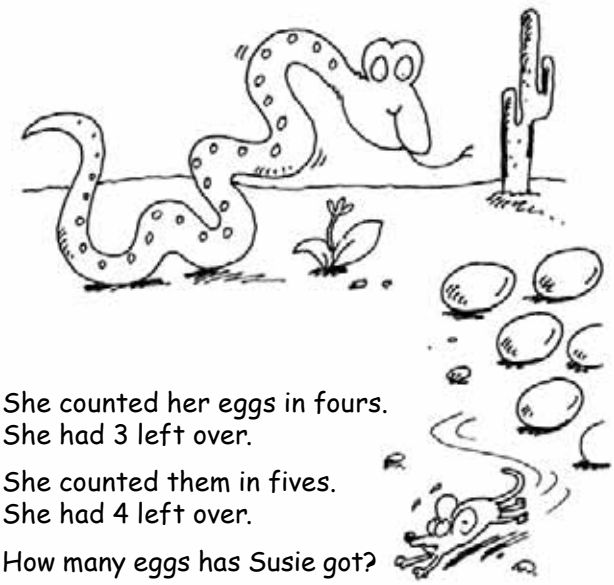
- Recall of multiples of 4, 5 and 10
- Know by heart addition facts up to 20
- Understand division as grouping
- Understand the idea of remainders
- Describe an array

Resources

- IWB version of the problem 'Susie the snake'
- Bag
- Snake puppet
- Counters/card eggs

Susie the snake

Susie the snake has up to 20 eggs.



She counted her eggs in fours.
She had 3 left over.

She counted them in fives.
She had 4 left over.

How many eggs has Susie got?

Main teaching activity

Show a version of Susie the snake to the children.
Set a context for the problem, e.g. Susie the snake has laid some eggs but she isn't very good at counting and isn't sure how many she has laid. To try and find out how many eggs she has, she lays them out in different ways.

Q. Can you describe the problem to a partner in your own words?
Q. What do we know? What have we got to find out?
Q. What might the number of eggs be? (9–19)
Q. What might the smallest number of eggs be?
Q. What couldn't the number be? (less than 9, 20 or more)
Ask the children to share their responses to the above with a partner, explaining their reasoning.
Q. Why do you think that?

This will help children to identify what they know about the possible range of numbers. Listen to the types of responses and ask further questions if needed.

At this point establish that we don't know whether there could be one or more answers.
Remind children about the fireworks problem and discuss what they did that helped them to know when they had found all of the possibilities.
Q. How will the way you solved that problem help you approach this problem?
Draw out the need for systematic recording to keep a check of their solutions and the use of listing to show that they could convince someone else that they had found all of the answers.
Q. Which number will you start with? Why?

Ask the children to work in pairs to solve the problem.

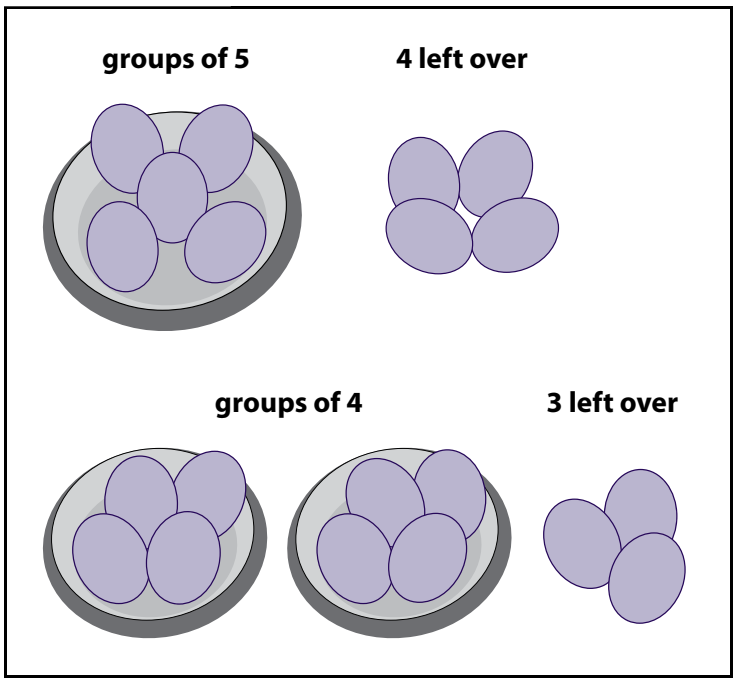
Paired work will encourage dialogue, develop reasoning and help children to feel more secure.

You may want to use an appropriate interactive whiteboard page showing a pile of 'eggs' ready to discuss the groupings and the relationships they're looking for. This may be particularly suitable for a small group of less able children. Encourage them to annotate and verbalise the groupings and decide whether they fulfil Susie's rules. Encourage these children to continue this method of reasoning for themselves.

Possible methods of recording:

Groups of 4 + 3 more	groups of 5 + 4 more
4 → 7	5 → 9
8 → 11	10 → 14
12 → 15	15 → 19
16 → 19	

Allow enough time for the children to talk about their ideas and methods of recording as they work through the problem.



Some children may need to list the multiples and then add the leftover eggs.

Drawing together

After the children have been working on the task for about 5 minutes, discuss one pair's systematic working or demonstrate this yourself.

Q. Could the answer be 9? Why not?

Try two groups of 5 and 4 more.

Q. Can we arrange these into groups of 4 and 3 more?

Q. Could 14 be the answer?

Ask the children to continue working.

Plenary

Ask several pairs of children to show the others their recording, and establish that Susie laid 19 eggs.

Q. What was important about your recording that helped you to be sure that you had solved the problem?

Q. When could you use this way of listing again?

'What if?' questions provide useful extensions to problems.

Q. What if Susie laid some more eggs? When she arranged them in 10s there were 2 groups of 10 and some left over. She arranged them in 3s and had 2 left over. She arranged them in 4s and still had 2 left over.

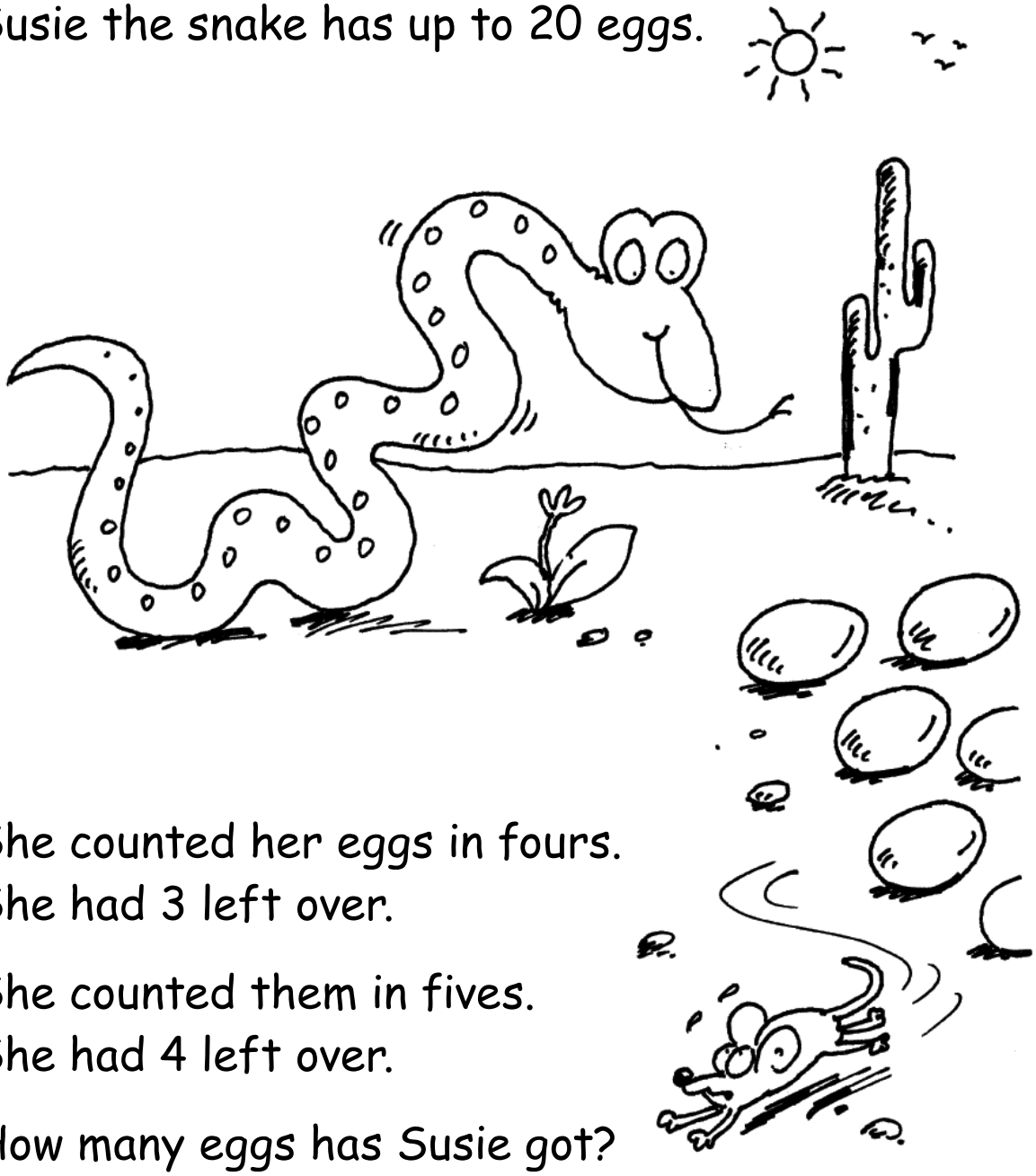
How do they manage this?
How appropriate are their examples?

Q. Could you think of a different problem similar to Susie the snake's, where you might make lists of multiples and then look for a pair that answers the problem?

Year 3, Lesson 2: resource sheet

Susie the snake

Susie the snake has up to 20 eggs.



She counted her eggs in fours.
She had 3 left over.

She counted them in fives.
She had 4 left over.

How many eggs has Susie got?

Year 4

Lesson 1: sheepdog trials

Primary Framework objectives

- Identify and use patterns, relationships and properties of numbers or shapes; investigate a statement involving numbers and test it with examples.
- Suggest a line of enquiry and the strategy needed to follow it; collect, organise and interpret selected information to find answers.

By the end of the lesson, children will be able to:

- solve a problem by checking possible solutions against given criteria;
- solve a problem by listing all possible answers;
- list possible answers in a systematic way.

Vocabulary

add sum even most/largest/biggest total least/fewest/smallest odd

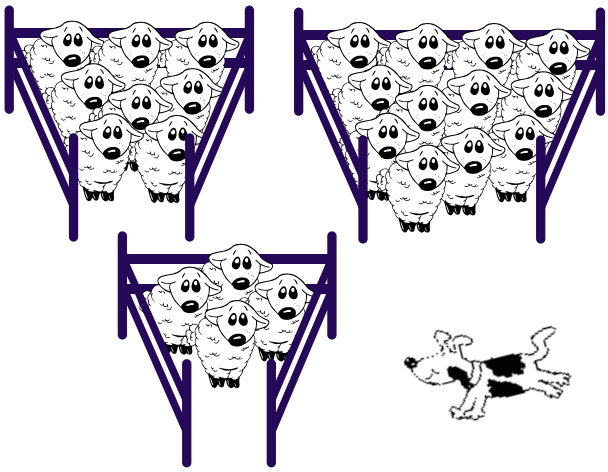
Necessary prior knowledge

- Recognise odd and even numbers
- Add and subtract mentally

Resources

- Resource sheet
- Image of 24 sheep to project or
- 24 magnetic counters and board

Sheepdog trials



A farmer and his dog enter a sheepdog trial. In this event his dog must shepherd 24 sheep into three pens of different sizes. Each pen must have a different even number of sheep. The largest pen must have the most sheep and the smallest pen must have the fewest sheep.

How many sheep might the dog try to get in each pen? Find as many different ways as you can.

Main teaching activity

Explain to the class that they are going to be solving a problem and that there is more than one correct answer to the problem. Explain that in the lesson you want the children to find as many of the possible answers that they can and that together you will be thinking about how they might organise their answers so that they know they have found all the answers.

The expected outcomes are more concerned with developing children's problem-solving skills than finding all the solutions. Children need to be able to make a list and check the possible solutions against criteria.

Give out the Resource Sheet and read the problem with the class.
Q. What is the question we are trying to answer?
Q. What are the important words and numbers?
Invite children to underline or highlight the important words and numbers on the sheet.
Encourage children to visualise the problem:
Q. What are the rules that the farmer must follow?
Record the rules for the class to refer to during the lesson:

Establishing the rules now will highlight to what extent children have understood the problem. A skill they need to solve the problem will be to ensure that solutions meet these criteria.

- Each pen must have a different number of sheep.
- Each pen must have an even number of sheep.
- The largest pen must always have the largest number of sheep and the smallest pen the smallest number of sheep. If you have an interactive whiteboard and appropriate clip art you could move 24 sheep around to satisfy the rules.

Q. Could a pen have three sheep? Why not?
Q. Could a pen have four sheep?
Q. Could each pen have four sheep? Why not?
Q. What is the smallest number of sheep a pen could have?
What is the largest number it could have?
Q. What numbers lie between 1 and 24?
Q. What other numbers of sheep could a pen have?
Ask children to list the numbers on their whiteboards.

Share the sheep/counters between the three pens (e.g. 1 in the first, 7 in the next and 16 in the next).
Q. Has the dog followed all the rules? Why/why not?
Ask the children to help you move the sheep/counters around so that they do satisfy the rules.
Write the solution on the board. Stress that this is only one possibility.
Q. Can you think of another possibility?
Ask children to write this solution on their whiteboards. Ask children to work in pairs to check that each child has followed all the rules.

Ask the children to work in pairs to find all possible answers. If children find all the solutions quickly, ask them to find solutions for a total of 21 sheep with each pen having an odd number of sheep.

Drawing together

Stop the class and encourage a child who has been working systematically to share their method with the class.

Q. What is the smallest number of sheep that the smallest pen can have? (2)
Q. How many sheep are left now? (22)
Q. What is the largest number of sheep that the biggest pen can have? (20)
Q. Can the largest pen have 22 sheep, or 20 sheep? (No, because that would leave the middle-sized pen with no sheep, or 2 sheep.)
Q. So how many sheep can each pen have? (2, 4, 18)
Q. What other possibilities can you find where the smallest pen only has 2 sheep?

Encourage the children to continue to solve the problem starting with the smallest number.

Drawing together

Ask the class for all the possibilities when the smallest pen has 2 sheep.
Using an interactive whiteboard, record these number sentences, and then move them around to show the order as shown here:
2 + 4 + 18
2 + 6 + 16
2 + 8 + 14
2 + 10 + 12

Moving the children's solutions around may help them to see that possibilities are all there, but that there is a pattern that can help them.

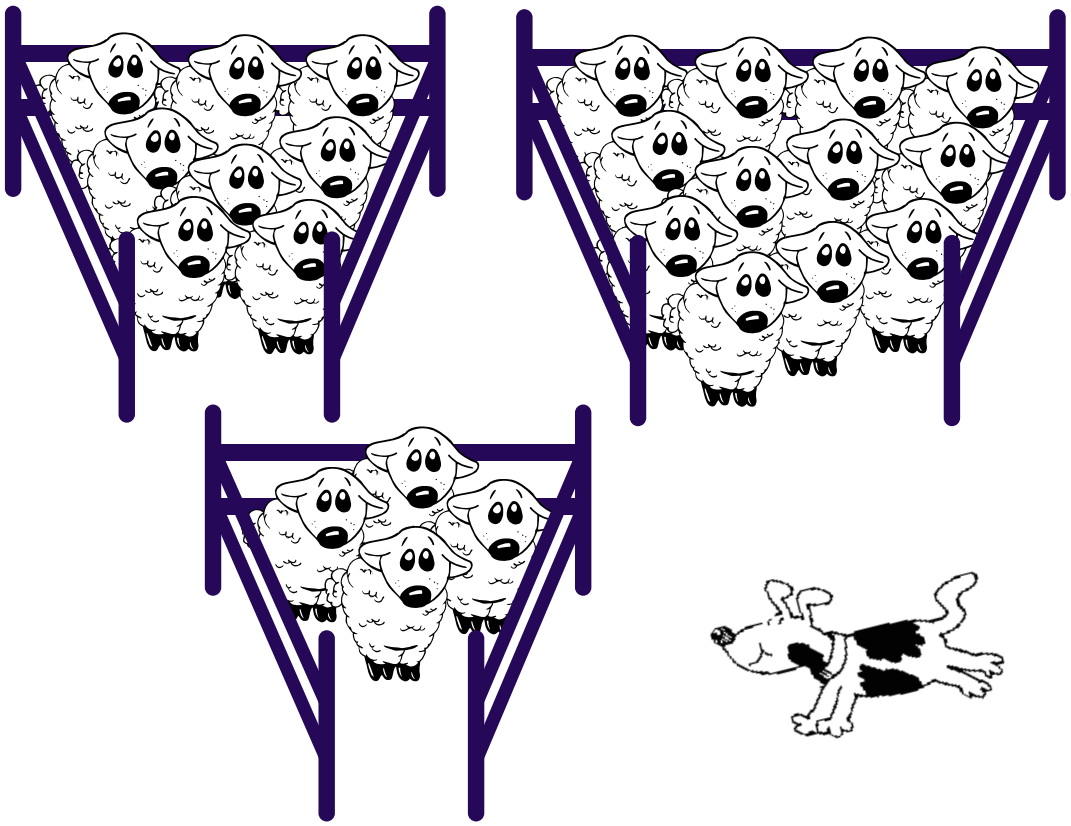
Q. Can you see a pattern? How does this help?
Q. Are there any more possibilities where the smallest pen has 2 sheep? How do you know? Explain why.
Q. What's the next smallest number of sheep the smallest pen could have? Encourage children to continue working systematically finding all possibilities.

Plenary

Ask the children for all the possibilities that they have found. Using an interactive whiteboard, you could ask children to record their solutions on it, and then move them around to form an ordered list.
Q. Do all our answers meet the rules?
Q. Have we got all of the possibilities? How do you know?
Q. How many answers are there to our problem? (7)
Q. Which strategies helped us to find all the answers?
Q. We started with the smallest number of sheep. What else could we have done?
Agree that you could have started with the greatest number of sheep in the largest pen.
Q. What would our list have looked like then?
Q. How does working systematically help us?

Year 4, Lesson 1: resource sheet

Sheepdog trials



A farmer and his dog enter a sheepdog trial. In this event his dog must shepherd 24 sheep into three pens of different sizes. Each pen must have a different even number of sheep. The largest pen must have the most sheep and the smallest pen must have the fewest sheep.

How many sheep might the dog try to get in each pen? Find as many different ways as you can.

Year 4

Lesson 2: three digits

Primary Framework objectives

- Identify and use patterns, relationships and properties of numbers or shapes; investigate a statement involving numbers and test it with examples.
- Suggest a line of enquiry and the strategy needed to follow it; collect, organise and interpret selected information to find answers.

By the end of the lesson, children will be able to:

- solve a problem by checking possible solutions against given criteria;
- solve a problem by listing all possible answers in a systematic way.

Vocabulary

digit sum add total difference

Necessary prior knowledge

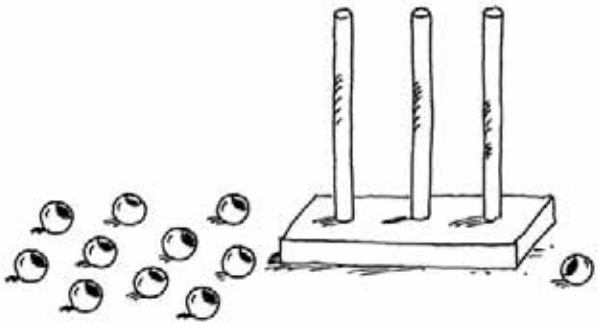
- Add and subtract mentally
- Recall of number facts to 20

Resources

- Individual whiteboards
- Resource sheet

Three digits

Imagine you have 25 beads.
You have to make a three-digit number on an abacus. You must use all 25 beads for each number you make.



How many different three-digit numbers can you make?
Write them in order.

Main teaching activity

Display the numbers 126, 512, 421, 460 on the board.

Q. What is a digit sum?
Q. Which of these three-digit numbers has a digit sum of 9?
Q. What is the digit sum of 123?
Q. What other three-digit numbers have a digit sum of 6?

Ask the children to answer on their whiteboards.

Remind the children of how working systematically helped them solve the sheepdog problem.

If children are working systematically, ask them to share their strategies, otherwise model working systematically starting with the smallest possible three-digit number as outlined here.

Q. How did we start the sheepdog problem? What number did we start with? Draw out that children started with the smallest or largest number and then had to think about what the remaining two numbers needed to be.
Q. What is the smallest possible three-digit number? (100)
Q. Does that total 6?
Q. If we start with a 1 in the hundreds place, what do the other two digits need to total? (5)

Write on the board: 105.

Q. Does this number have a digit sum of 6?
Q. What other numbers could we put in the tens and ones places?

Write the following list on the board:

105
114
123
132
141
150

Which children spot the pattern in the tens and ones columns?

Writing the list in this way should encourage the children to work systematically themselves when tackling the problem.

Q. Can you spot any pattern in how I have recorded these possibilities?
Q. Can you explain it to your partner?

Discuss how we could continue finding digit sums that total 6.

Q. What might we do next? What number could we put in the hundreds place? Then what do we need to find out about the remaining two digits?

Draw out starting with 2 and thinking of pairs of numbers that total 4.

Explain that we are not going to continue with this problem but will move on to a different problem.

We are not trying to solve this particular problem here. The focus is on modelling and practising problem-solving skills that are needed to tackle the three digits problem.

Present the problem to the class, providing a context, for example:

I have a new car and I would like to have a personalised number plate. The letters will be SLG and I then need to choose a three-digit number. My lucky number is 25 so I would like to choose three digits that have a total of 25. What three digits could I choose?

Q. How might you tackle this problem?
Q. Could working systematically help you?

Q. How will you record all the possibilities?
Q. How will you know that you have all the possible three-digit numbers?
Allow the children to start to solve the problem working independently. Some children may find it useful to have digit cards (children will need more than one set to solve the problem).

One strategy is to choose a starting number and then work out what the remaining two numbers need to total. Some children will use a different strategy: they may have discovered that the digits in 799 have a total of 25 and then simply rearrange the digits to give other possibilities. Look out for this and share with the rest of class.

Drawing together

Q. Has anyone found any three-digit numbers where the digits add up to 25 that have a 1 in the hundreds place? Why not?
Establish that with a 1 in the hundreds place, the remaining two digits would need to total 24 and that it is not possible to have two single digits that total 24.
Q. What other numbers can you not have in the hundreds place?
Q. Why not?
Ask the children to continue, to find all possibilities. If children quickly find all six solutions set a new challenge: How many four-digit numbers can you find where the digits add up to 25 but have a 9 in the thousands place? This generates far more possibilities and should encourage the children to work and record in a systematic way.

Plenary

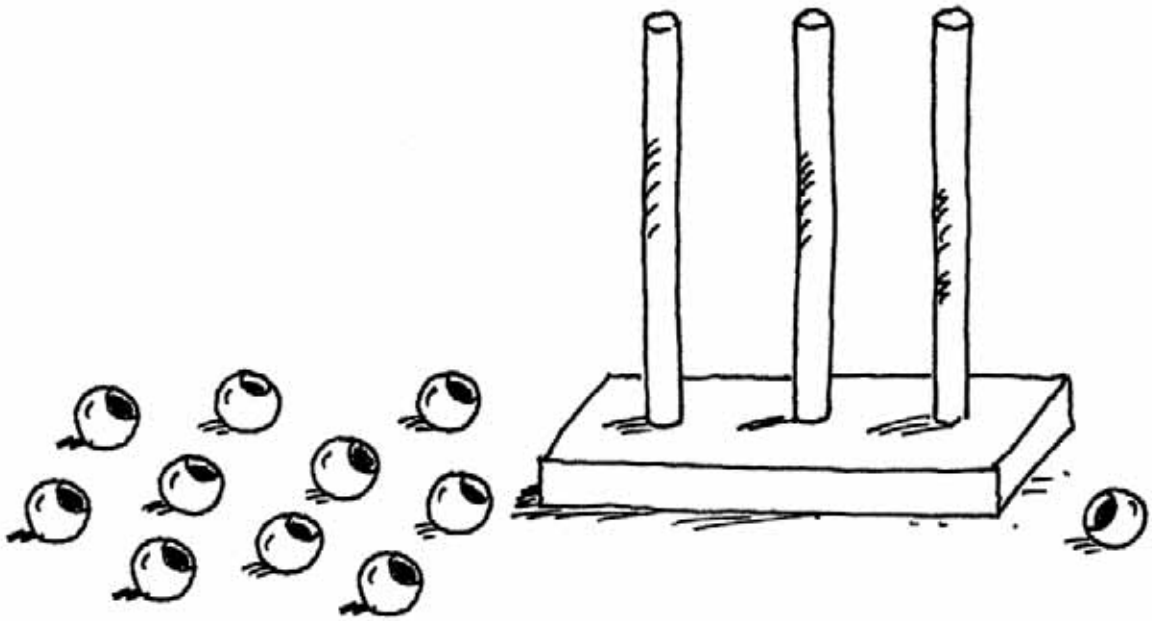
Q. What could the numbers on my new number plate be?
Q. How do you know that you have all the possibilities?
Q. What strategies were useful?
Q. If my lucky number was smaller than 25 would I have more or less choice of digits? Why/Why not?
Q. If my number had to have four digits, what number might I choose? Ask the children to write a possible solution on their whiteboards.
Q. How did you work it out?
Write one solution on the board (1987).
Q. If you know that the digits in 1987 total 25, how could you use this to find other numbers where the digits have a total of 25?

This question will highlight several strategies. Some children may rearrange the digits to suggest 7891. Some children will look at the total of the last two digits (15) and suggest numbers such as 1996.

Year 4, Lesson 2: resource sheet

Three digits

Imagine you have 25 beads.
You have to make a three-digit number on an abacus. You must use all 25 beads for each number you make.



How many different three-digit numbers can you make?
Write them in order.

Year 5

Lesson 1: double scoop ice-creams

Primary Framework objectives

- Explore patterns, properties and relationships and propose a general statement involving numbers or shapes; identify examples for which the statement is true or false.
- Plan and pursue an enquiry; present evidence by collecting, organising and interpreting information; suggest extensions to the enquiry.

By the end of the lesson, children will be able to:

- find all possibilities by working systematically;
- begin to see how working systematically can help explain reasoning.

Vocabulary

investigate find all pattern justify strategy

Necessary prior knowledge

- Finding combinations

Resources

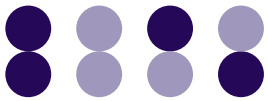
- Crayons
- Coloured counters
- Number facts Interactive Teaching Program (ITP)
- Interactive whiteboard software

Double scoop ice-creams

How many two-scoop ice-cream cones can we make with two flavours?

Main teaching activity

Begin the lesson by posing this problem:
Q. How many different two-scoop ice-cream cones can we make with two flavours?
Ask the children to work in pairs to discuss this and jot down their answers. Make sure that there are two different coloured counters or crayons available for those children who wish to use them.



S V S V or SS VV SV VS
S V V S

It is important to model that there are different ways of recording including the use of colour.

Drawing together

Discuss with the children their findings. Leave a range of different ways of recording to display to the rest of the class.
Discuss with the children their different ways of recording their answers and whether SV and VS are the same or different. Establish that for the purposes of this problem they are different, because the cones are made with scoops one on top of the other, and so there would be four different cones.

Ask the children to discuss the following questions with their partner:
Q. What if there were 2 flavours but 3 scoops of ice-cream?
Q. Can you predict how many different cones there would be?
Collect some responses and discuss how they might record their findings. Then ask the class to carry on with the investigation.

Drawing together

Ask the children to share their solutions in a whole-class discussion, making sure that the following questions are addressed:

Q. How can you be sure that you have found all the different combinations?
Q. How can the way you record your findings help you?
Q. Are there different ways of being systematic with your recording?
Share their different responses and establish that there are 8 different cones possible.

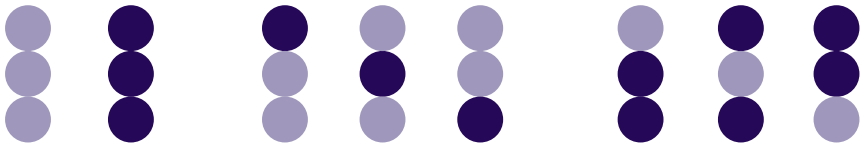
Compare the two problems and how systematic recording is vital if we are to prove that we have found all of the combinations of flavours. If possible, use children's work that illustrates this, but if not model the recording yourself.

2 scoops:

1 flavour		**1 strawberry/ 1 vanilla**		**Total**
S	V	S	V	
S	V	V	S	
2		2		**4**

3 scoops:

1 flavour		**1 strawberry / 2 vanilla**			**2 strawberry / 1 vanilla**			**Total**
V	S	S	V	V	V	S	S	
V	S	V	S	V	S	V	S	
V	S	V	V	S	S	S	V	
2		3			3			**8**



Ask the children to discuss with their partners:

Q. What patterns and relationships can you see in these recordings?
Q. How is this recording the same or different to your own?
Q. How effective is this recording in ensuring that all combinations of ice-cream cone have been found?

Make sure that you encourage the children to use their recordings to justify their claims that they have found all possibilities.

Ask the children:

Q. What if there were 2 flavours and 4 scoops per cone?
Q. Can we use our previous answers to predict the result?

Discuss with them how identifying a group and then finding all the possibilities within that group is an effective approach. Ensure that they understand that there are different ways of grouping possibilities.

Ask the children to discuss with their partner an effective method of recording and then to work together to find all the possible combinations.

Plenary

Establish that there were 16 different cones altogether and that working systematically was necessary to prove that all combinations were identified.

Q. Was it necessary to list all of the combinations of flavours within each group or could it have been possible to work out the total for one group, and then use this information to find the total for another?

We also want the children to realise that being systematic not only ensures that all combinations are identified, but also makes the process of finding all possibilities more efficient.

4 scoops:

1 flavour	1 straw / 3 van	3 straw / 1 van	2 straw / 2 van	Total
V S	S V V V	V S S S	V V V S S S	
V S	V S V V	S V S S	V S S S V V	
V S	V V S V	S S V S	S S V V V S	
V S	V V V S	S S S V	S V S V S V	
2	4	4	6	**16**

Through discussion, draw out that 1 vanilla/3 strawberry is the reverse of 1 strawberry/3 vanilla and so both groups would have the same number of cones.

Q. What if we had 5 scoops; can you predict how many combinations there would be?
Q. Which groups would have the same number of cones in them?

For the final question you do not want the children to make lists but to identify criteria for groups that could be generalised, and so the children begin to see a shortcut to list making.

Year 5

Lesson 2: treasure hunt

Primary Framework objectives

- Explore patterns, properties and relationships and propose a general statement involving numbers or shapes; identify examples for which the statement is true or false.
- Plan and pursue an enquiry; present evidence by collecting, organising and interpreting information; suggest extensions to the enquiry.

By the end of the lesson, children will be able to:

- find all possibilities by working systematically;
- begin to see how working systematically can help explain reasoning.

Vocabulary

investigate find all pattern justify strategy

Necessary prior knowledge

- Recording information in a table

Resources

- Crayons
- Counters
- Interactive whiteboard software

Treasure hunt

Jed and Jake are pirates.
Between them they have
three precious jewels:
a ruby (R), a diamond (D) and an emerald (E).

Complete the table.
Show what jewels each pirate could have.

Jed	R							
Jake	D E							

Main teaching activity

Display the recording from the previous lesson and ask the children to discuss with their partners what they had learned during that lesson that might help them solve other problems. Take feedback.

Tell the children that they are now going to use what they learned to solve another problem.

Q. There are two pirates, Jed and Jake. They have an emerald, a diamond and a ruby between them. Which jewels might each pirate have?

Make sure that you stress the point that looking for groupings that can be reversed is an efficient way of using listing as a problem-solving strategy, as they did when finding combinations of ice creams.

Establish that each pirate must have at least one jewel. Ask the children to discuss with their partner how they are going to start the problem.

Q. What recording will you need to use?

Q. How will you decide to group your recordings?

Ask the children to find the solution to the problem. Some children may wish to use counters and crayons while other children will wish to represent their recording with letters.

Look for children who have used a table or have found the answers for one pirate and realised that they can use this to find the answers for the other pirate. This problem differs from the ice-cream problem in that they are not being asked to find a number of combinations but to list which jewels each pirate might have, so a table might be a useful way of organising their recording.

Drawing together

Look for at least two pairs of children who have recorded their work systematically and logically but have made different decisions about the way they devised their groupings or set their recording out.
Ask them:

Q. Can you explain the thinking behind your solution? e.g.:

Jed	Jake
R E	D
R D	E
D E	R

'I knew that one pirate had to have two jewels while the other pirate had one. So once I had found all the solutions for Jed having two and Jake having one, I could swap over the names at the top of the columns and then I would have all the solutions for Jake having two and Jed having one.'

This is an important teaching point. Once children are able to list systematically, they need to understand that information can be transferred and other lists or combinations can be deduced.

Q. What if the two pirates now had four jewels between them? They have to have at least one each.

Ask the children to discuss with their partner how they are going to tackle this problem.

Some children might find it helpful to think about these questions:

Q. Would creating a table be useful?
Q. Would it be helpful to create more than one table?

Ask the children to solve the problem.

Plenary

The focus of this plenary is to help the children evaluate their decisions, strategies and recordings and to communicate their reasoning. The following questions would be useful in helping children to develop and reflect on their learning.

Q. Can you explain your strategy?
Q. How did you check that you had found all of the solutions?
Q. Can you think of another strategy that might have worked?
Q. Could there be a quicker way of doing this?
Q. Can you make up a similar problem that would be easy to solve using those skills you have been practising?
Q. If you were solving a similar problem what would you do the same? What would you do differently?
Q. What have you learned today?

You do not have to ask all these questions in the plenary, but you will need to decide which are appropriate for developing your children's thinking and reasoning.

Year 6

Lesson 1: King Arnold

Primary Framework objectives

- Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols, e.g. the cost of *c* pens at 15 pence each is 15*c* pence.
- Suggest, plan and develop lines of enquiry; collect, organise and represent information, interpret results and review methods; identify and answer related questions.

By the end of the lesson, children will be able to:

- understand strategies to record the solutions to a visual problem in a systematic way.

Vocabulary

systematic predict pattern

Necessary prior knowledge

- Some experience of systematic recording strategies for problems (see Year 4 and Year 5)

Resources

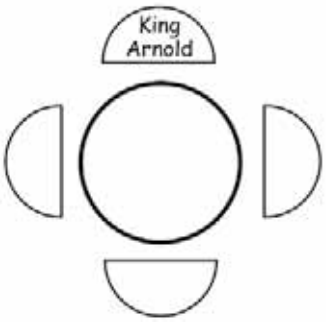
- Self-adhesive labels or sticky notes
- Large sheets of paper and felt pens
- Resource sheet showing the King Arnold problem
- Solution sheets

King Arnold

King Arnold sits at a Round Table.

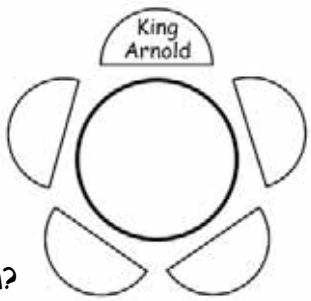
There are 3 empty seats.

In how many different ways can 3 knights sit in them?



What if there are 4 empty seats?

In how many different ways can 4 knights sit in them?



Main teaching activity

Read the King Arnold problem to the children and explain that the focus of the lesson will be to look at a range of recording strategies and to discuss their strengths and weaknesses.

Ask children to work in groups of four and sit on four chairs arranged as in the problem. Give each child a label and ask them to label one member of their group King Arnold and the others Knights A, B and C. Ask them to act out the problem and help each other to move the three knights. You could use a digital camera to capture possible seating arrangements, and manipulate these images to form a list of possibilities.

After five minutes or so, ask the children to discuss in their groups how they might move the knights systematically. If the children are struggling you may need to ask them to discuss whether pictures, lists or tables might help. Taking digital photographs is another possibility.

The focus of this part of the lesson is to try and understand the problem. Role-play should help children to understand and see how they might go about it.

Q. How might you record it? How have you recorded similar problems previously?

Next ask all the kings to meet together, all Knight As in another group, Knight Bs in another and Knight Cs in a fourth group. Ask them to share how their groups are thinking they might move systematically. They should record the ways on a large sheet of paper.

Ask the children to go back to their original groups and try out a way of finding all the possibilities. Each group should record these on a large piece of paper.

Drawing together

Look at each large piece of paper in turn and ask the children from other groups to describe what they think it says.

Q. How do you know you've found all the possibilities?
Q. Did your recording system help you to know how you had them all?
Q. What would happen if there were four knights and four empty seats? Would your recording system still work?

The focus here is children evaluating the different recording strategies.

Ask the children to continue working in fours but to imagine four empty seats and four knights, and to record all the possible seating arrangements.
Encourage them to decide first on a recording system. If the children are struggling you could show them the start of one recording system and ask them to complete it. If some children do this quickly, you could ask them to consider 5 knights and discuss any difficulties this presents.

Q. Are you going to change your recording system? Why/why not?

Plenary

Q. What recording strategies have we used?
Q. What were the advantages and disadvantages of each? Discuss the time taken to record the possibilities, the ease with which others could interpret the record and how easy it was to make predictions.
Q. How easy was the strategy to use when the problem extended to four knights?
Q. If you drew a table for three knights, how could/did you adapt this for four knights?

Seat 1	Seat 2	Seat 3
a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a

Draw out that adding an extra column for the 4th seat would be helpful.
Knight D could sit here and so there are six possible arrangements with the knight in this place.

It is useful for children to see that once they have generated one set, they can predict how many seating arrangements there will be without listing them all.

Q. What might we do next?
Draw out that we could swap Knight A and D thus creating another set of seating arrangements.

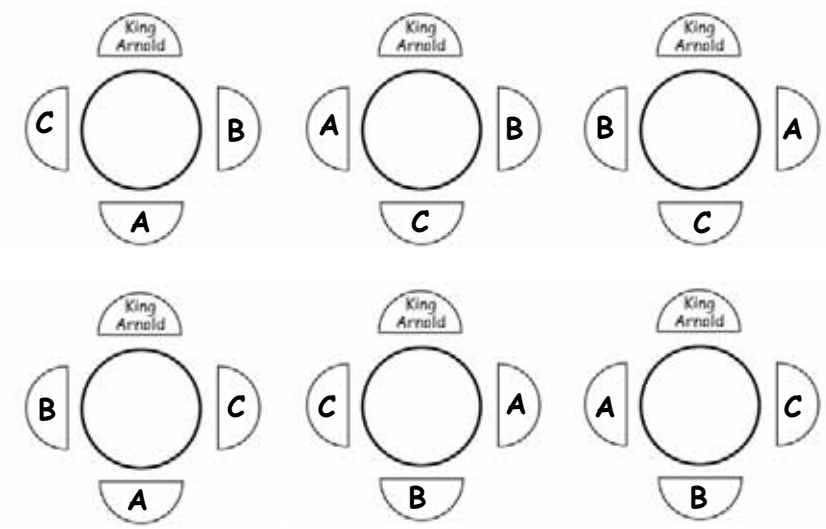
Q. So how many arrangements would there be altogether? Do we need to record them all?
Q. What have we learned?
Q. What is useful for us to remember for another time?

Year 6, Lesson 1: solution (1 of 2)

Possible strategies for recording solutions for King Arnold

Knights lettered A B C Seats numbered 1 2 3

Pictorial Representation



Symbolic Representation

Systematic strategy . . . Move A through each place, move B and C systematically in each of these cases

1A	2B	3C
1A	2C	3B
1B	2A	3C
1C	2A	3B
1B	2C	3A
1C	2B	3A

Systematic strategy . . . Putting each knight in turn opposite King Arnold

1	2	3
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

Year 6, Lesson 1: solution (2 of 2)

King Arnold solutions ... What happens when King Arnold moves as well, i.e. there are four people on the move?

There are 24 possibilities now.

Build on the strategy used for three knights.
Fix each knight in turn at the top of the table.
Pull out the strong effect of the pattern.

1	2	3	4
A	B	C	D
A	B	D	C
A	C	B	D
A	D	B	C
A	C	D	B
A	D	C	B
B	A	C	D
B	A	D	C
B	C	A	D
B	D	A	C
B	C	D	A
B	D	C	A
C	A	B	D
C	A	D	B
C	B	A	D
C	D	A	B
C	B	D	A
C	D	B	A
D	A	B	C
D	A	C	B
D	B	A	C
D	C	A	B
D	B	C	A
D	C	B	A

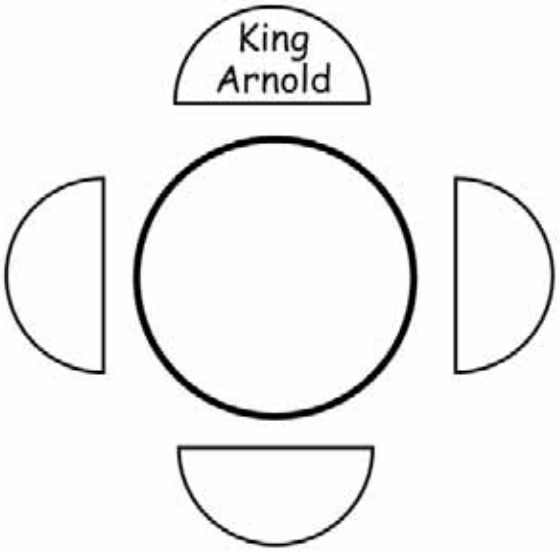
Year 6, Lesson 1: resource sheet

King Arnold

King Arnold sits at a Round Table.

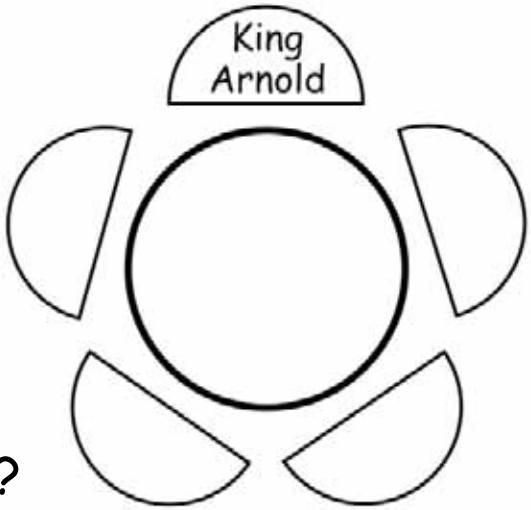
There are 3 empty seats.

In how many different ways can 3 knights sit in them?



What if there are 4 empty seats?

In how many different ways can 4 knights sit in them?



Year 6

Lesson 2: four by four

Primary Framework objectives

- Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols, e.g. the cost of c pens at 15 pence each is 15c pence.
- Suggest, plan and develop lines of enquiry; collect, organise and represent information, interpret results and review methods; identify and answer related questions.

By the end of the lesson, children will be able to:

- understand strategies to record the solutions to a visual problem in a systematic way.

Vocabulary

identical reflection area rotation shape systematic square

Necessary prior knowledge

- Identical, rotation, reflection
- Systematic recording strategies for problems (see Year 4 and Year 5)

Resources

- Squared paper
- Scissors
- Tracing paper
- Mirrors
- Individual whiteboards
- **Area* Interactive Teaching Program (ITP)
- Appropriate software with rotation facility

* You will need to display the Area ITP. Children will need either squared paper or computers with the Area ITP loaded on them.

Four by four

This 4 by 4 grid is divided into two identical shapes. Each shape has the same area.

How many more ways can you find of dividing the grid into two identical shapes by colouring the squares two different colours?

Main teaching activity

Read the 'Four by four' problem with the children. Check their understanding of the problem. Show two solutions:

Q. Are both of these valid solutions? Why not?
Draw out that although the shapes have the same area, the two parts in the second grid are not identical.

Ask children to work in pairs to produce examples on squared paper. The *Area ITP* is useful for children to find solutions on.

Drawing together

Q. How do you know if two parts are identical?
Q. How are you finding different solutions? Do you have a system?
Ensure that children can see that row 1 and row 4 are inverses, i.e. if row 1 has 3 purple squares and 1 blue square then row 4 must have 1 purple square and 3 blue squares.

Ask children to now go and see if they can find all the possible solutions to the problem. Children working quickly could be challenged further:
Q. Would you be able to do this with a 5×5 square? Explain your answer. (The grid needs to have even number of rows and columns to avoid half squares.)

Drawing together

Take different examples of solutions.
Q. How can we organise our answers?
Q. What ways did we have of organising our work in the King Arnold lesson?
Remind pupils of fixing one variable and manipulating others.

Show children a way of being systematic by fixing the colour in one row and varying the answers in next row (see solution).

Q. Which of these solutions are repeats?
Agree that rotations and reflections are repeats.
Q. How can we check for repeats?
Encourage children to use tracing paper or mirrors as appropriate to check. Ask the children to organise their solutions systematically and then discard repeats.

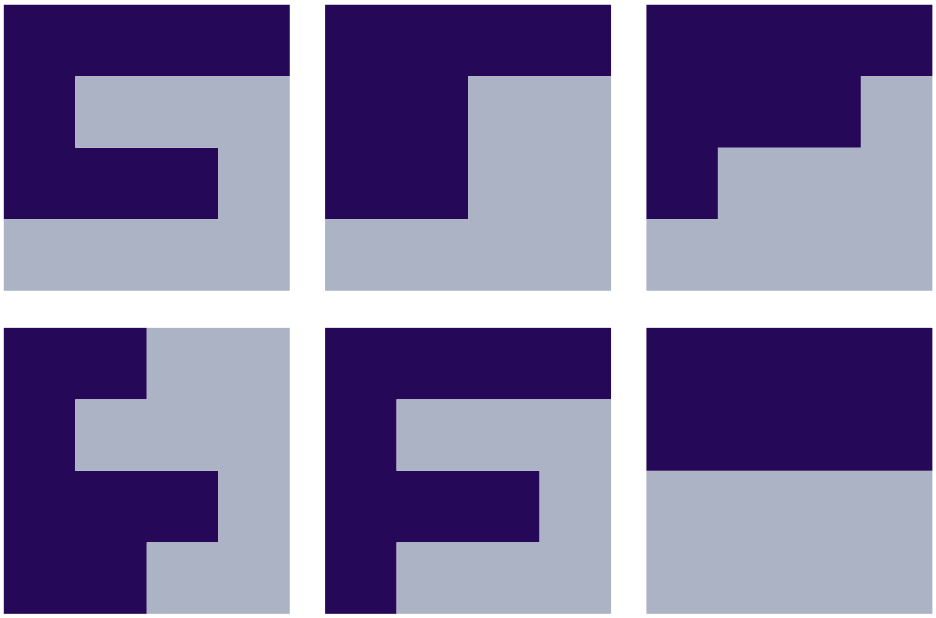
Plenary

Q. How many solutions have you found?
Agree that there are six different solutions, discarding reflections and rotations.

Show the class some of the children's early work.
Q. Where did we start? How organised was our work?
Q. What helped us to become more organised?
Q. What is the value of becoming more organised?
Q. What is useful for us to remember for another time?

Emphasise that the strategies of being systematic can be applied to visual problems.

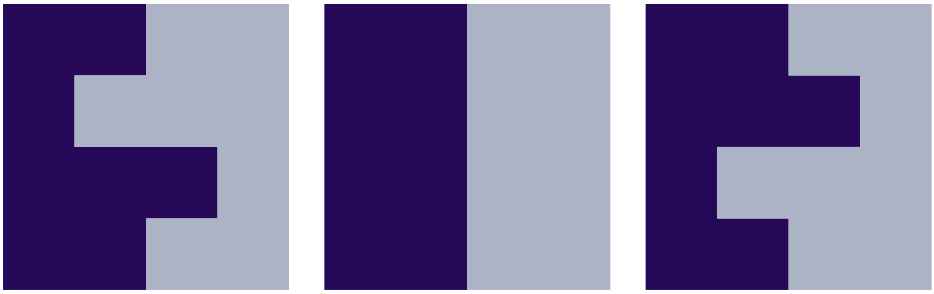
The six solutions are:



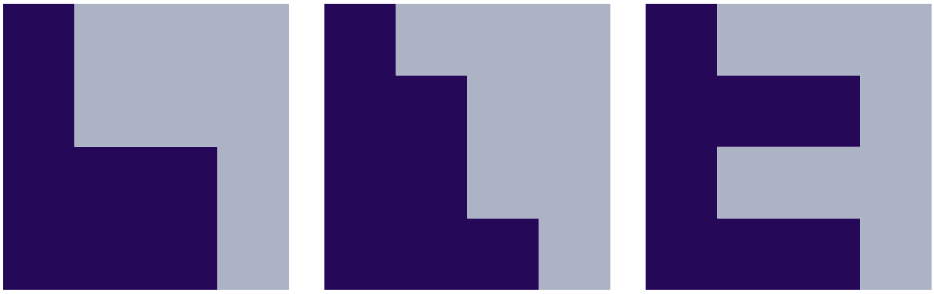
Four blue in first row

Three blue in first row

Two blue in first row



One blue in first row



Audience: EYFS, KS1 and KS2 teachers, mathematics subject leaders and LA mathematics consultants.
Date of issue: 07-2010
Ref: **00346-2010PDF-EN-01**

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